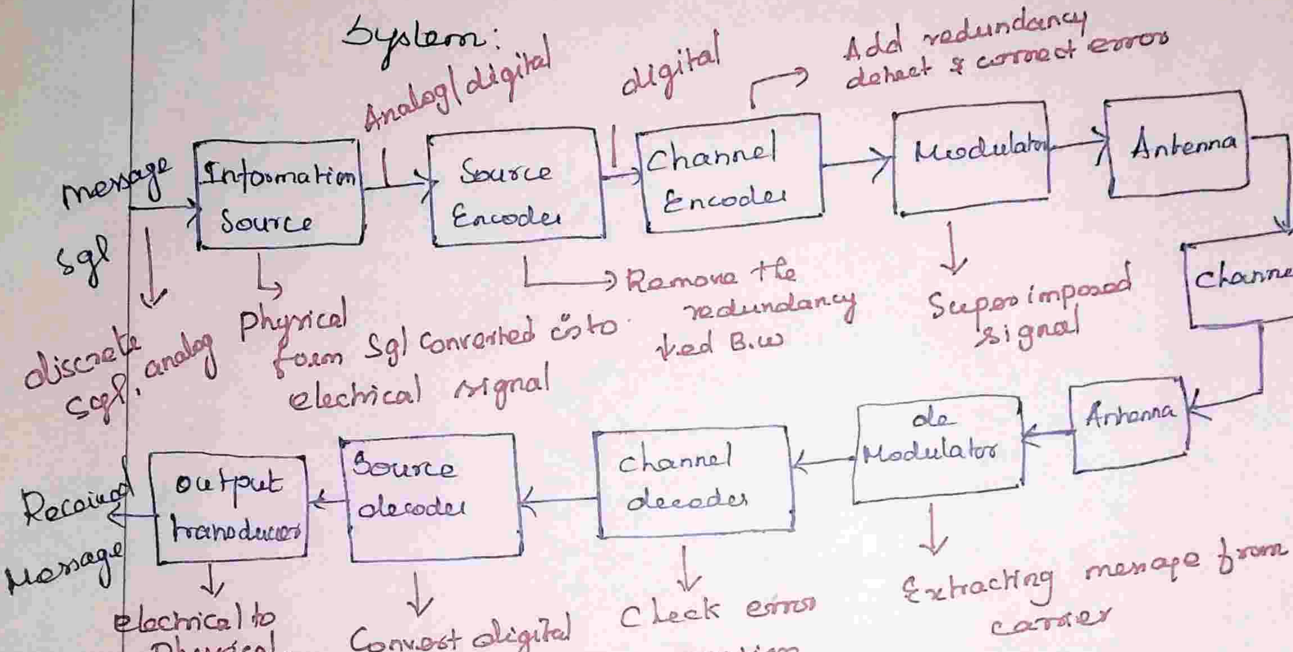
SUBJECT: DIGITAL COMMUNICATION

UNIT: I

INFORMATION THEORY

I. Block diagram of Digital Communication System:



Source:

The source can be an analog signal.

Example: A sound signal.

Input Transducer:

The transducer which takes a physical input and converts it to an electrical signal.

Example: Microphone.

This block also consists of an analog to digital converter where a digital signal is needed for further processes.

A digital signal is generally represented by a binary sequence.

Source Encoder

→ The source encoder compresses the data into minimum number of bits. This process helps in effective utilization of the bandwidth

→ It removes the redundant bits. ie unnecessary excess bits.

Channel Encoder:

→ The channel encoder, does the coding for error correction and detection. During the transmission of the signal, due to the noise in the channel, the signal may get altered and hence to avoid this, the channel encoder add some redundant bits to the transmitted data. These are error correcting data bits

Digital Modulator

→ The signal to be transmitted is modulated here by a carrier.

→ The signal is also converted to analog from digital sequence in order to make it travel through the channel or medium.

Channel

→ The channel or medium allows the analog signal to transmit from the transmitter end to receiver end.

Digital demodulator

→ This is the first step at the receiver end. The received signal is demodulated as well as converted again from analog to digital.

→ The signal gets reconstructed here.

Channel Decoder

→ The channel decoder, after detecting the sequence, does some error corrections. The distortions which might occur during the transmission, are corrected by adding some redundant bits.

→ This addition of bits helps in the complete recovery of the original signal.

Source decoder.

→ The resultant signal is once again digitized by sampling and quantizing so that the pure digital output is obtained without the loss of information.

→ The source decoder recreates the source o/p.

Output Transducer.

→ Output transducer converts the signal into the original physical form which was at the input of the transmitter.

→ It converts the electrical signal into physical o/p. (Ex. loud speaker)

Output signal

This is the output which is produced after the whole proces

Example The sound signal received.

Discrete Memory less Source:

→ A source in which each system is generated independently.

→ Information is a source of a communication. whether it is an analog or digital.

Definition of DMS:

A Source from which the data is being emitted at discrete instant of time and if the emitted data is independent of previous data.

Ex face Book

Because information is emitted only at discrete instant of time.

Characteristics of DMS:

↳ List of symbols.

↳ probability of symbol

↳ specification of rate of generating symbols by source.

Information Content of DMS:

↳ should be proportional to the uncertainty of an outcome.

↳ Information contained in independent outcomes should add.

Properties of DMS:

(a) (i) $I(x_i) = 0$ for $P(x_i) = 1$

(ii) $I(x_i) \geq 0$

(iii) $I(x_i) > I(x_j)$ if $p(x_i) < p(x_j)$

(iv) $I(x_i, x_j) = I(x_i) + I(x_j)$

$x_i, x_j \rightarrow$ independent.

Unit of $I(x)$ = <u>bit</u>

$I(x) = 0 \quad p(x) = \infty$

Entropy:

Entropy canbe defined as a measure of the average information content per source symbol.

$$H = \frac{\text{Total Information}}{\text{No. of message}}$$

Consider M Message ($M_1, M_2 \ldots M_m$) are emitted from the source with propabilites $P_1, P_2, \ldots P_m$ and L sequence is transmitted.

$M_1 \rightarrow$ Number of Messages.

No of m_1 messages in L sequence $= P_1 L$

No. of m_2 Messages in L sequence $= P_2 L$

No. of m_k messages in L sequence $= P_k L$

Amount of information carried by m_1, m_1 messages is

$$I_1 = P_1 L \log_2 (1/P_1)$$

|||ly

$$I_k = P_k L \log_2 (1/P_k).$$

Let I_{total} is the total information carried by 'L' message sequence, therefore.

$$I_{total} = I_1 + I_2 + I_3 + \cdots I_m$$

$$= P_1 L \log_2 \left(\frac{1}{P_1}\right) + P_2 L \log_2 \left(\frac{1}{P_2}\right) + \cdots + P_k L \log_2 (1/P_k)$$

$$I_{total} = \sum_{k=1}^{M} L P_k \log_2 (1/P_k)$$

$$\text{Entropy} = \frac{I_{total}}{\text{Length of sequence}} = \frac{\sum_{k=1}^{M} L P_k \log_2 (1/P_k)}{L}$$

$$\boxed{\text{Entropy}(H) = \sum_{k=1}^{M} P_k \log_2 (1/P_k) \text{ bits/symbol}}$$

Condition of occurrance of events

* If the event has not occurred there is a condition of uncertainty.
* If the event has just occurred there is a condition of surprise.
* If the event has occurred, a time back there is a condition of having some information.

Properties of Entropy:

(i) The entropy of sure or impossible events is zero.

Proof

for impossible event

probability $P_X = 0$

$$H = \sum_{k=1}^{M} P_k \log_2 (1/P_k)$$

$P_k \rightarrow 0$.

$\boxed{H = 0}$

Sure events

probability $P_x = 1$

$$H = \sum_{k=1}^{M} P_k \log_2 (1/P_k)$$

$P_k = 1$

$$H = \sum_{k=1}^{M} 1 \log_2 (1/1)$$

$\log_2 1 = 0$

$\boxed{H = 0}$

(ii) If there are M no of equally likely messages then the entropy of the messages source is $\log_2 M$

Proof

Given that M equally likely messages therefore, the probability of occurrence of each message is $1/M$.

i.e $\frac{1}{P_1} = \frac{1}{P_2} = \frac{1}{P_3} \ldots \frac{1}{\frac{1}{M}}$

$P_1 = 1/M = P_2 = P_3 \ldots P_K$

Entropy $(H) = \sum_{k=1}^{M} P_k \log_2 (1/P_k)$

$P_1 \log_2 (1/P_1) + P_2 \log_2 (1/P_2) + P_3 \log_2 (1/P_3)$

$\ldots P_k \log_2 (1/P_K)$

Substitute the values of $P_1 = P_2 = P_3 = P_k = \frac{1}{M}$.

$\frac{1}{M} \log_2 \left(\frac{1}{1/M} \right) + \frac{1}{M} \log_2 \left(\frac{1}{1/M} \right) +$

$\frac{1}{M} \log_2 \left(\frac{1}{1/M} \right)$

(6)

$$\frac{1}{M} \log_2 \left(\frac{1}{1/M}\right)^M$$

$$\frac{1}{M} \log_2 (M)^M = \frac{M}{M} \log_2 M$$

$$\boxed{H = \log_2 M}$$

(iii) If M Number of Messages emitted by a Source then the upperbound on entropy is given as $H_{max} = \log_2 M$

Proof

(a) To prove this consider basic property of natural logarithm ie $\ln x \le x - 1$ — (1).

Consider two probability distributions

$P_1, P_2, P_3 \ldots P_K$, $q_1, q_2, q_3 \ldots q_K$. on the alphabet.

$X = \{x_1, x_2 \ldots x_K)$

Consider

$$\sum_{k=1}^{M} p_k \log\left(\frac{q_k}{p_k}\right)$$

$$= \sum_{k=1}^{M} \frac{p_k \log_{10}\left(\frac{q_k}{p_k}\right)}{\log_{10} 2}$$

$$= \sum_{k=1}^{M} \frac{p_k \log_{10}\left(\frac{q_k}{p_k}\right)}{\log_{10} 2} \times \frac{\log_{10} e}{\log_{10} e}$$

$$\frac{\log_{10} e}{\log_{10} 2} \sum_{k=1}^{M} \frac{p_k \log_{10}\left(\frac{q_k}{p_k}\right)}{\log_{10} e}$$

$$\log_2 e \sum_{k=1}^{M} \frac{p_k \log_{10}\left(\frac{q_k}{p_k}\right)}{\log_{10} e} \quad \therefore \log_2^e = 1$$

$$\log_2 e \sum_{k=1}^{M} p_k \log_e\left(\frac{q_k}{p_k}\right)$$

$$\log_2 e \sum_{k=}^{M} p_k \ln\left(\frac{q_k}{p_k}\right) \quad \text{——— (2)}$$

Consider $x = \frac{q_k}{p_k}$ then

$$\ln\left(\frac{q_k}{p_k}\right) \leq \frac{q_k}{p_k} - 1$$

From eq 2

$$\sum_{k=1}^{M} p_k \log_2\left(\frac{p_k}{q_k}\right) \le \log_2 e \sum_{k=1}^{M} p_k\left(\frac{q_k}{p_k}-1\right)$$

$$\le \log_2 e \sum_{k=1}^{M} p_k\left(\frac{q_k-p_k}{p_k}\right)$$

$$\le \log_2 e\left[\sum_{k=1}^{M} q_k - \sum_{k=1}^{M} p_k\right]$$

$$\boxed{\text{Here } \sum_{k=1}^{M} q_k = \sum_{k=1}^{M} p_k = 1}$$

$$\sum_{k=1}^{M} p_k \log_2\frac{q_k}{p_k} \le 0.$$

$$\sum_{k=1}^{M} p_k \log_2 q_k \quad \cancel{=} \quad \sum_{k=1}^{M} p_k \log_2 \quad \le 0$$

$$\sum_{k=1}^{M} p_k \log_2 q_k - \sum_{k=1}^{M} p_k \log_2 \frac{1}{p_k} \le 0$$

$$\sum_{k=1}^{M} p_k \log_2 q_k - \sum_{k=1}^{M} p_k \log_2\left(\frac{1}{p_k}\right) \le 0$$

$$\sum_{k=1}^{M} p_k \log_2\left(\frac{1}{p_k}\right) \le \sum_{k=1}^{M} p_k \log_2\left(\frac{1}{q_k}\right)$$

$$H \le \left[p_1 \log_2\frac{1}{q_1} + p_2 \log_2\frac{1}{q_2} + \cdots \; p_M \log_2 \frac{1}{q_M}\right]$$

M - messages if I consider these equally likely messages then

$$P_1 = P_2 = \frac{1}{M}, \quad \frac{1}{q} \; q_1 = q_2 \cdots = \frac{1}{M}$$

$$H \leq \left[\frac{1}{M}\log_2\frac{1}{1/M} + \frac{1}{M}\log_2\frac{1}{1/M} \cdots + \frac{1}{M}\log_2\frac{1}{1/M}\right]$$

$$H \leq \frac{1}{M}\left[\log_2 M + \log_2 M + \cdots + \log_2 M\right]$$

$$H \leq \frac{1}{M}\log_2(M)^M$$

$$H \leq \frac{\cancel{M}}{\cancel{M}}\log_2 M \Rightarrow H \leq \log_2 M$$

$$\boxed{H \leq \log_2 M} \longrightarrow \text{proved.}$$

$$\boxed{H_{max} \leq \log_2 M}$$

Information Rate, joint Entropy, Marginal Entropy and Conditional Entropy:

Information Rate (R).

It is the product of signal rate (r) and Entropy (H) is called Information rate (R).

Units of $R = \frac{\text{Symbol (message)}}{\text{second}} \cdot \frac{\text{bit}}{\text{message}}$

Information rate = bits/seconds.

Joint Entropy and Conditional Entropy of Marginal Entropy.

Let us consider the joint occurrence of two events x_i, y_j then we have 4 probabilities

* Probability of X is $P(x_i)$
* probability of Y is $P(y_j)$
* Joint probability of $P(X, Y)$ is $p(x_i, y_j)$
* The transition or Conditional probability $P(x_i/y_j)$. $p(y_j/x_i)$

If we add all the joint probabilities for fixed x_i we get

$$P(x_i) = \sum_{j=1}^{M} P(x_i, y_j)$$

If we add all the joint probabilities for the fixed y_j we get $P(y_j) = \sum_{i=1}^{M} p(x_i, y_j)$

Joint Entropy:

If $p(x_i, y_j)$ is the joint probability of occurrence then $H(X,Y) = \sum_{i=1}^{M} \sum_{j=1}^{M} p(x_i, y_j) \log_2 \frac{1}{p(x_i, y_j)}$

Marginal Entropy:

When the Entropy of individual event is evaluated from joint probabilities of the events it is called "Marginal Entropy"

$$H(X) = \sum_{i=1}^{M} P(x_i) \log_2 \frac{1}{P(x_i)}$$

Marginal Entropy of X is $\sum_{i=1}^{M} \left(\sum_{j=1}^{M} P(x_i, y_j) \log_2 \frac{1}{P(x_i)} \right)$

Marginal Entropy of Y is $\sum_{i=1}^{M} \sum_{j=1}^{M} P(x_i, y_j) \log_2 \frac{1}{P(y_j)}$

$$H = \sum_{k=1}^{M} P_k \log_2 \frac{1}{P_k}$$

$$H[X] = \sum_{i=1}^{M} P(x_i) \log_2 \frac{1}{P(x_i)}$$

Mutual Information

It is defined as the amount of Information transferred where x_i is transmitted and y_j is received.

$$M.I = I(x_i, y_j) = \log_2 \left[\frac{P(x_i|y_j)}{P(x_i)}\right] \quad \text{—(1)}.$$

Average Mutual Information:

Amt of source information gained per received Symbol.

$$I(X,Y) = \sum_{i=1}^{m} \sum_{j=1}^{M} P(x_i, y_j)\, I(x_i, y_j)$$

$$I(X,Y) = \sum_{i=1}^{n} \sum_{j=1}^{M} P(x_i, y_j) \log_2 \left[\frac{P(x_i, y_j)}{P(x_i)}\right]$$

Properties of Mutual Information.

(i) Mutual Information is symmetric.

$$I[X,Y] = I[Y,X]$$

(ii) Mutual Information is always positive

$$I[X,Y] \geq 0.$$

(iii) Mutual Information may be expressed as Entropies

$$MI = H[X] - H[X/Y]$$

$$= H[Y] - H[Y/X]$$

(iv) Mutual Information is related to joint Entropy.

$$I[X,Y] = H(X) + H(Y) - H[X,Y]$$

$$I[X,Y] = H[X] + H[Y] - \underbrace{H[X,Y]}_{\text{joint Entropy}}.$$

Discrete Memoryless Channel:

Discrete → x_k, y_k are discrete.

Memoryless → present y_k depends upon present x_k only (not on past elements)

↳ No memory elements.

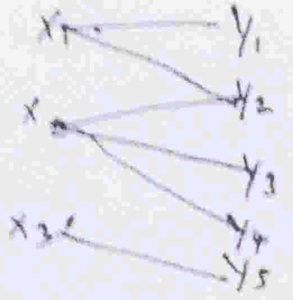
(9)

Channel Matrix

$$\frac{Y(k)}{X(k)} \quad \begin{array}{c|ccccc} & y_1 & y_2 & y_3 & \cdots & y_n \\ x_1 & y_1/x_1 & y_2/x_1 & y_3/x_1 & \cdots & y_n/x_1 \\ x_2 & y_1/x_2 & y_2/x_2 & y_3/x_2 & \cdots & y_n/x_2 \\ x_3 & y_1/x_3 & y_2/x_3 & y_3/x_3 & \cdots & y_n/x_3 \\ x_m & y_1/x_m & y_2/x_m & y_3/x_m & & \end{array}$$

(1) Lossless channel

(2) Deterministic channel.



$\Sigma R_i = 1$

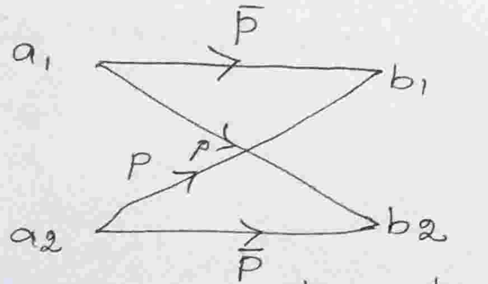
So every column has atleast one non-zero term.

(3) Noise channel:

- Each Row of the channel Matrix $Y(k)/X(k)$ corresponds to a fixed channel I/p and each column of the matrix corresponds to a fixed channel o/p.

Binary symmetric channel.

A symmetric channel is defined as which has two input and two o/p, is called as Binary symmetric channel.



$$P(B/A) = \begin{array}{c|cc} & b_1 & b_2 \\ \hline a_1 & \bar{p} & p \\ a_2 & p & \bar{p} \end{array}$$

$p + \bar{p} = 1$ or $\bar{p} = 1 - p$.

As is symmetric channel

$$C = \log_2 s - h, \quad h = \sum_{j=1}^{s} P_j \log_2\left(\frac{1}{P_j}\right)$$

$$S = 2, \quad h = \bar{p} \log_2 \frac{1}{p} + p \log_2 \frac{1}{p}$$

$$\therefore C = \log_2^2 - \bar{p} \log_2 \frac{1}{p} + p \log_2 \frac{1}{p}$$

$$C = 1 - \bar{p} \log_2 \left(\frac{1}{p}\right) + p \log_2 \left(\frac{1}{p}\right)$$

$$\boxed{C = 1 - h}$$

$$C = \log_2 s - h$$

$$= \log_2 2 - h \Rightarrow \boxed{C = 1 - h}$$

Channel Capacity: - Harley, shannon law

Channel Capacity:

Maximum rate of information from transmitter to receiver (in channel). It is denoted by C'

→ Information rate (R) $<$ C (Channel Capacity)

→ Information rate (R) $>$ C (Channel Capacity)

$R < C$: Transmission without error even in presence of noise.

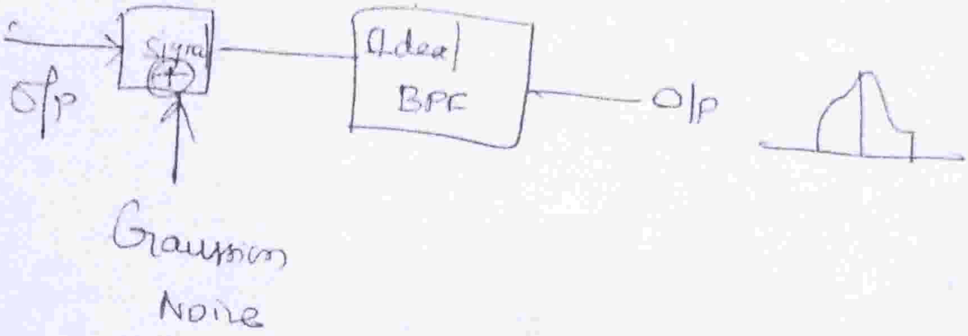
$R > C$: Transmission without error is not possible with original signal in the presence of noise.

To get noise free transmission, the error detection and correction codes are added to original signal. (in presence of noise)

∴ Signal length ↑ed → circuit complexity

Channel Capacity = $C = B \log_2 (1 + S/N)$ bits/second

B → Bandwidth



V_s = rms value of signal voltage

$\left[V_{rms} = \frac{V_p}{\sqrt{2}} \right]$

Signal power {Resistance (R)} $= V_s^2/R = V_s^2$

Noise power {Resistance (R) $= V_N^2/R \Rightarrow V_N^2$

$S = V_s^2 \Rightarrow V_s = \sqrt{S}$

$N = V_N^2 \Rightarrow V_N = \sqrt{N}$

m(no of levels) $= \sqrt{\frac{S+N}{\sqrt{N}}} \Rightarrow \frac{1+\sqrt{S}}{N}$

$I = \log_2(m) = \log_2\left(\sqrt{1+S/N}\right)$

$= \frac{1}{2}\log_2(1+S/N)$ for k bits/symbol

$C = \frac{k}{2} \log_2 (1 + S/N)$ | Condn $S < N$

SNR ↑ed ⇒ C ↑ed ⇒ $C = \infty$

$C = \infty$, $S/N = \infty$

B ↑ed, ↑C → $B \to \infty$

If channel Bandwidth $= B$.

$C = B \log_2 (1 + S/N)$

Noise power spectral density $\eta/2 = \eta/2$

Noise power $(N) = \eta B$.

$C = B \log_2 \left(1 + S/\eta B\right)$

$= \log_2 \left(1 + S/\eta B\right)$

$C = \frac{\eta}{S}\left(\frac{BS}{\eta}\right) \cdot \log_2 \left(1 + S/\eta B\right)$

~~$C = B \log_2 (1 + S/N)$~~ $\boxed{C = B \log_2 (1 + S/N)}$

$C = \frac{\eta B}{}$ $\quad C = \frac{S}{\eta} \log_2 \left(1 + \frac{S}{\eta B}\right)^{\frac{\eta B}{S}}$

$$\boxed{\lim_{x \to 0} (1+x)^{1/x} = e}$$

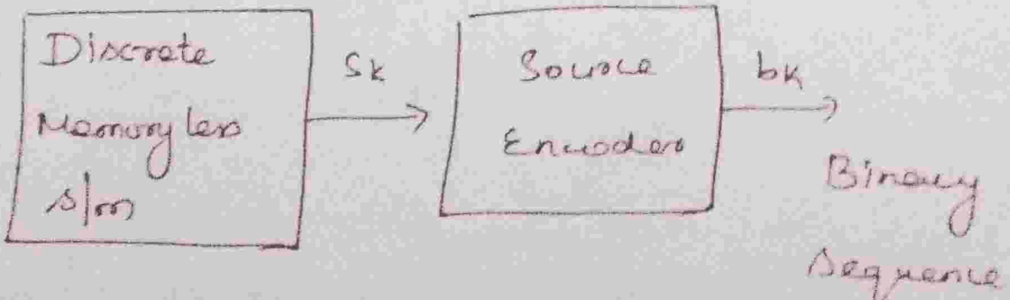
$$\boxed{C = \frac{S}{\eta} \log_2 e}$$

Source Coding theorem:

The discrete memoryless source produces the code that has to be represented efficiently. It is one of the important problems in communications.

Consider telegraphy where Morse code is used. The alphabets are represented by marks and spaces.

If letter E is taken that is used mostly, it is denoted by "" whereas the letter Q that is used rarely is denoted by "–, –"



Here $s_k \rightarrow$ discrete memoryless source o/p

$b_k \rightarrow$ source encoder o/p.

rep by 0's and 1's.

→ The encoded sequence is easily decoded at the receiver

Consider the source has an alphabets with k different symbols and k^{th} symbol S_k, occurs with probability P_k where $k = 0, 1, \ldots k-1$

S_k is assigned with binary code word, by the encoder having length l_k, is measured in bits.

→ The average codeword length 'L' of the source encoder is defined as

$$L = \sum_{k=0}^{k-1} P_k l_k.$$

$L \rightarrow$ rep. avg no of bits per source symbol

$L_{min} \rightarrow$ Minimum possible value of L

Code efficiency $\eta = \frac{L_{min}}{L}$

$$\boxed{L \geq L_{min} \qquad \eta \leq 1}$$

The source encoder efficiency when $\eta = 1$

L_{min} has to be determined

$L_{min} = H(S)$

$\eta = \frac{H(S)}{L}$ → Source encoder efficiency in terms of Entropy.

SHANNON - FANO Encoding

↳ Used to reduce the redundancy

↳ to use B w spectrum efficiency

Algorithm

1. The messages are first written in the order of decreasing probability.
2. Then divide the message set into two most equiprobable subset X and Y.
3. The message of 1'st set X is given bit '0' and message in the 2nd subset is given bit 1.

4. The procedure is now applied for each set separatly till end.

5. Finally we get the code word for respective symbol.

6. calculation.

$$\text{Efficiency } (\eta) = \frac{H}{\hat{H}}$$

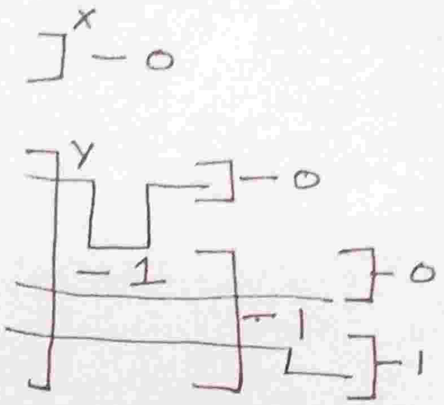
$$H \rightarrow \text{Entropy} = \sum_{i=1}^{M} P_i \log_2\left(\frac{1}{P_i}\right)$$

$$\hat{H} \Rightarrow \sum_{i=1}^{M} P_i \eta_i$$

Example: Find the code words occuring in the Probability $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\}$ for Symbols S_1, S_2, S_3 & S_4. find efficiency and redandency of code.

Soln:

Symbol	Prob
S_1	$1/2$
S_2	$1/4$
S_3	$1/8$
S_4	$1/8$



Codeword	length (n)
0	1
1 0	2
1 1 0	3
1 1 1	3

$\rightarrow$ efficiency $(\eta) = \dfrac{H}{\hat{H}}$

where, H = Entropy = $\sum_{i=1}^{n} P_i \log_2 \left(\frac{1}{P_i}\right)$

$$\hat{H} = \sum_{i=1}^{n} P_i\, n_i$$

$\rightarrow$ Redendency

$R_e = 1 - \eta$

Efficiency $\eta = \frac{H}{\hat{H}}$

$$H = \sum_{i=1}^{4} P_i \log_2 \left(\frac{1}{P_i}\right)$$

$$= \frac{1}{2} \log_2 2 + \frac{1}{4} \log_2 4 + \left(\frac{1}{8} \log_2 8\right) \times 2$$

$$= \frac{1}{2} + \frac{1}{2} \frac{3}{4}$$

$= 1.75$ bits / Symbol.

$$\hat{H} = \sum_{i=1}^{4} P_i n_i$$

$$= \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 \times 2$$

$= 1.75$ bits / symbol.

$$\therefore \eta = \frac{H}{\hat{H}} = \frac{1.75}{1.75} = 1$$

- Redendancy $R_e = 1 - \eta$

$= 1 - 1 = 0$

Huffman coding

Steps

1. The source symbols are arranged in order of decreasing probability. Then the two of lowest probability are assigned bits 0 and 1.

2. Then combine last two symbols and move the combined symbol as high as possible

3. Repeat the above step until end.

4. code for each symbol is found by moving backward.

5. Calculation.

$$\text{efficiency } \eta = \frac{H}{L \log_2 r}$$

$r \rightarrow$ binary $r = 2$ (0,1)

ternary $r = 3$ (0,1,2)

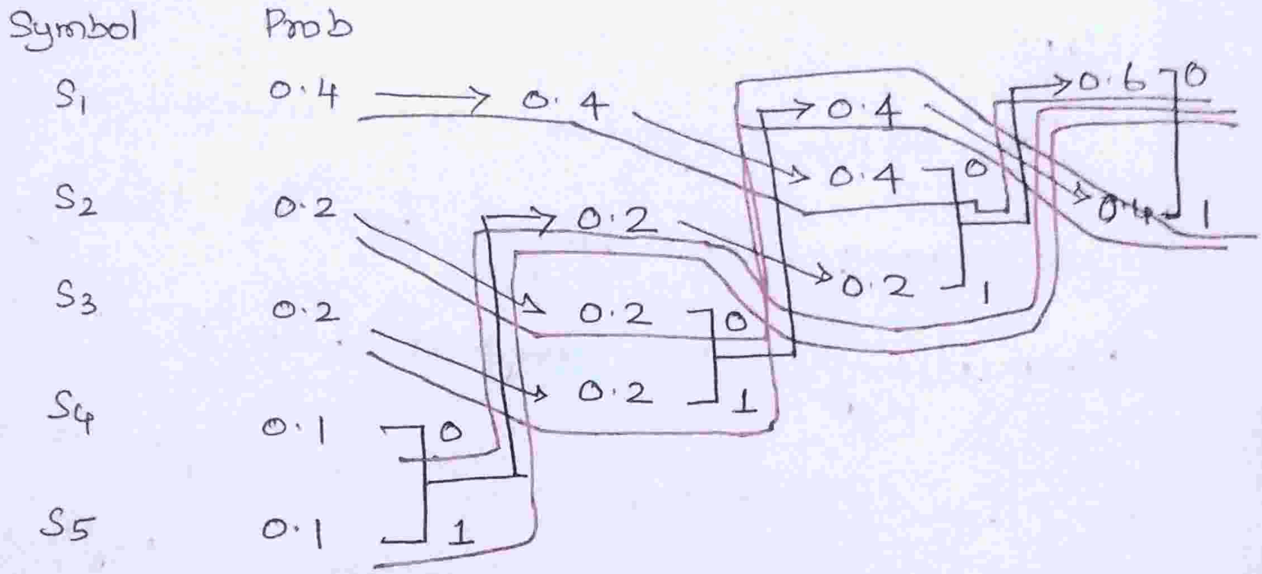
$$L = \text{Avg. codeword} = \sum_{i=1}^{n} P_i n_i$$

Variance

$$\sigma^2 = \sum_{i=1}^{n} P_i (n_i - L)^2$$

$$H = \sum_{i=1}^{n} P_i \log_2 \left(\frac{1}{P_i}\right)$$

Example: Alphabet with prob = $\{0.4, 0.2, 0.2, 0.1, 0.1\}$ For Symbols $\{S_1, S_2, ----, S_5\}$. Find Huffman Codes and also find efficiency & Varience.



Symbol	Codeword	length
S_1	0 0	2
S_2	1 0	2
S_3	1 1	2
S_4	0 1 0	3
S_5	0 1 1	3

– Entropy

$$H = \sum P_i \log_2 \left(\frac{1}{P_i}\right)$$

$$= 0.4 \log_2 \left(\frac{1}{0.4}\right) + 2 \times 0.2 \log_2 \left(\frac{1}{0.2}\right) + 2 \times 0.1 \log_2 \left(\frac{1}{0.1}\right)$$

$$= 2.1216 \text{ bits/symbol.}$$

$- L = \Sigma p_i n_i$

$= 2 \times 0.4 \times 2 (2 \times 0.2) + 3 \times 0.1 \times 2$

$= 2.2$ bits/symbol

$- \eta = \dfrac{H}{L \log_2 r} = \dfrac{2.1216}{2.2 \times \log_2 2} = 96.4\%$

$- \sigma^2 = \Sigma p_i (n_i - L)^2$

$= 0.4 (2 - 2.2)^2 + 2 \times 0.2 (2 - 2.2)^2 + 2 \times 0.1 (3 - 2.2)^2$

$= 0.16$

↑ As low as possible.

UNIT: II

WAVEFORM CODING AND REPRESENTATION.

Prediction filtering and DPCM:

Prediction filtering:

Linear prediction is a mathematical operation where future values of a discrete time signals are estimated as a linear function of previous samples.

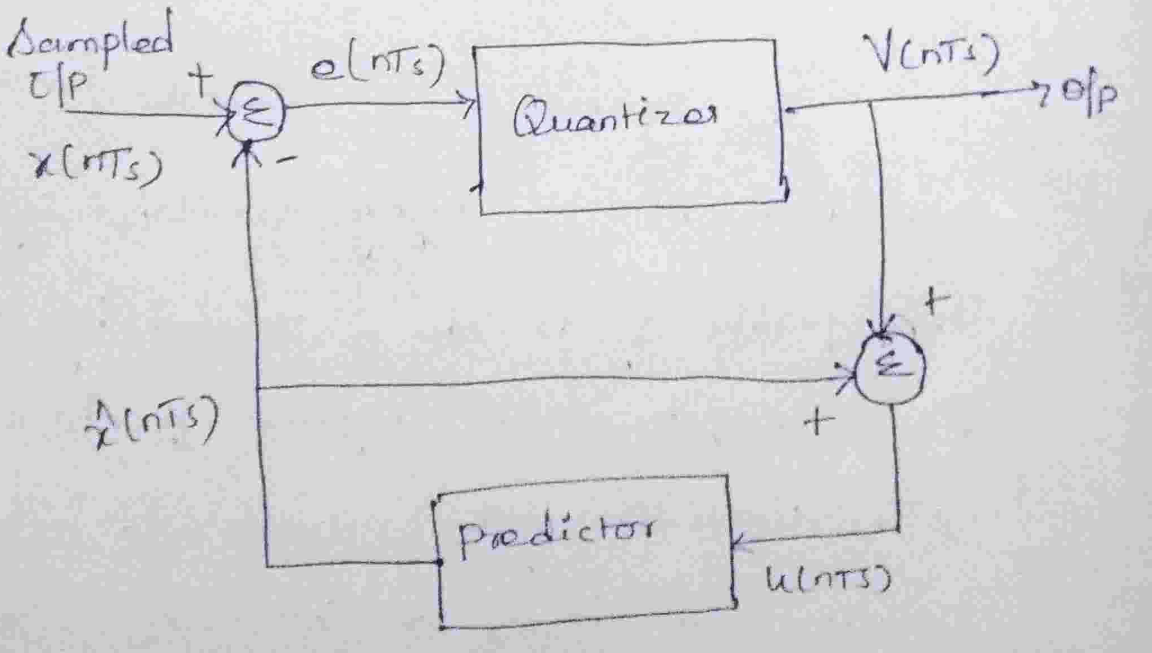
Use of Prediction filter is DPCM.

The DPCM s/m is suitable for digitalization and transmission of highly correlated signals. This quality of the s/m is provided by a Prediction filter in the negative feedback loop.

→ The prediction filter estimate the actual sample value based on one or more previous samples of input signal. (source).

DPCM transmitter:

In PCM, the redundant information leave behind. To process this redundant information and to have a better o/p, it is a wise decision to take a predicted sampled value, assumed from its previous o/p. and summarize them with the quantized values. Such a process is called as Differential PCM technique.



→ The DPCM transmitter consists of quantizer and predictor with two summer circuit.

→ The signal at each point are named as

$x(nT_s)$ is the sampled input

$\hat{x}(nT_s)$ is the predicted sample

$e(nT_s)$ is the difference of sampled i/p and predicted o/p. often called as prediction error.

$V(nT_s)$ is the quantized o/p.

$u(nT_s)$ is the predictor i/p which is actually the summer o/p of the predictor o/p and the quantizer o/p.

→ The predictor produces the assumed samples from the previous o/p of the transmitter circuit.

→ The i/p to the predictor is the quantized versions of the input signal $x(nT_s)$

→ The quantized o/p is represented as

$$V(nT_s) = Q\left[e(nT_s)\right]$$

$$= e(nT_s) + q(nT_s)$$

$q(nT_s) \longrightarrow$ quantization error.

Predictor i/p is the sum of quantizer o/p and predictor o/p.

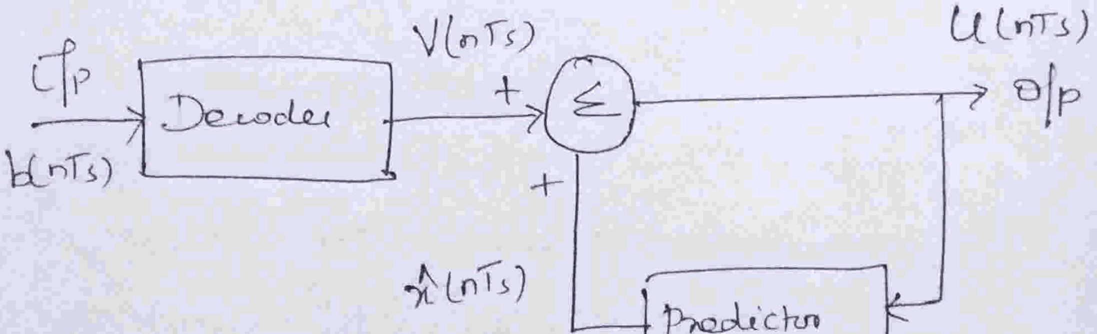
$$u(nT_s) = \hat{x}(nT_s) + v(nT_s)$$

$$u(nT_s) = \hat{x}(nT_s) + e(nT_s) + q(nT_s)$$

$$u(nT_s) = x(nT_s) + q(nT_s)$$

→ The same predictor ckt is used in the decoder to reconstruct the original input.

DPCM Receiver:



DPCM Receiver consists of

decoder

Predictor

summer ckt.

→ The notation of the signals is the same as the previous one.

→ In the absence of noise, the encoded receiver input will be the same as the encoded transmitter O/P.

→ The predictor assumes a value, based on the previous outputs. The I/P given to the decoder is processed and that O/P is summed up with the O/P of the predictor to obtain a better O/P.

→ The sampling rate of the signal should be higher than the Nyquist rate, to achieve better sampling.

→ If this sampling interval in DPCM is reduced considerably, the sample to sample Amplitude difference is very small.

→ The difference is 1-bit quantization.

→ Step size is very small i Δ (delta)

Delta Modulation

The type of modulation, where the sampling rate is much higher and in which the stepsize after quantization is of a smaller value Δ, such a Modulation is termed as delta Modulation.

Features of Delta Modulation:

→ An Over sampled i/p is taken to make full use of the Signal correlation.

→ The quantization design is simple.

→ The i/p sequence is much higher than the Nyquist rate.

→ Quality is moderate

→ design of modulator and demodulator is simple.

→ stair case Approximation of o/p waveform

→ Step size is very small.

→ bit rate can be decided by the user

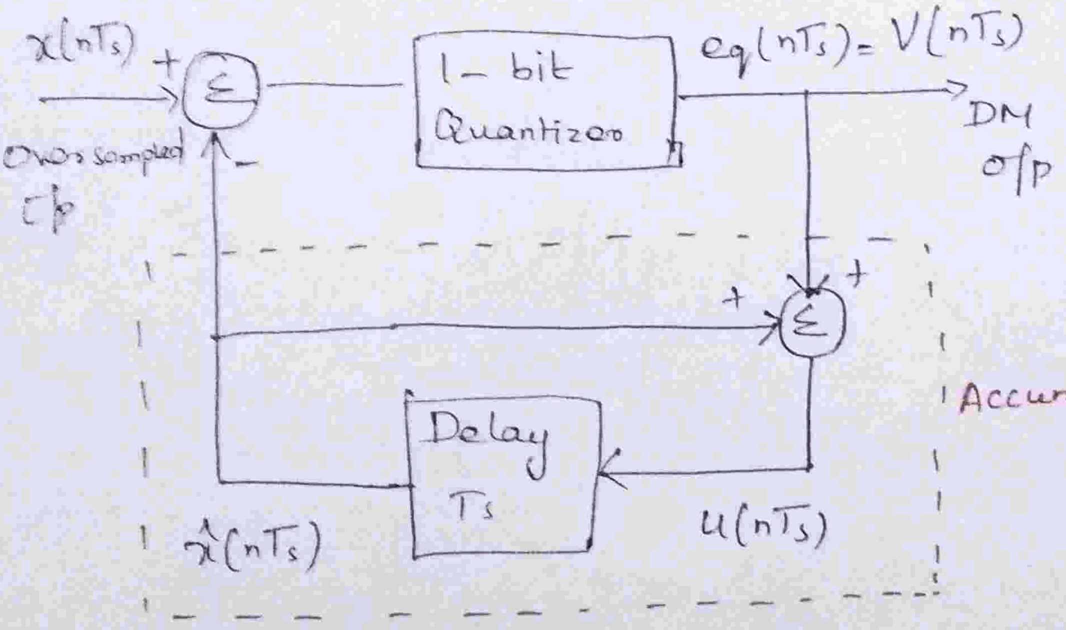
Delta Modulation is simplified form of DPCM.

→ It is 1-bit DPCM scheme.

→ Sampling interval is Reduced.

→ Signal correlation will be Higher.

Delta Modulator:



→ Delta Modulator comprises of a 1-bit quantizer and a delay circuit along with two summer circuits.

The predictor circuit DPCM is replaced by a simple delay circuit in DM.

$x(nT_s)$ = over sampled i/p

$e_p(nT_s)$ = Summer o/p and quantizer i/p.

$e_q(nT_s)$ = quantizer o/p = $V(nT_s)$

$\hat{x}(nT_s)$ = o/p of delay circuit.

$u(nT_s)$ = i/p of delay circuit.

$$e_p(nT_s) = x(nT_s) - \hat{x}(nT_s) \quad \text{—— (1)}$$

$$= x(nT_s) - u[n-1]T_s$$

$$= x(nT_s) - \left[\hat{x}(n-1)T_s\right] + V[n-1]T_s\Big] \quad \text{—— (2)}$$

$$V(nT_s) = e_q(nT_s) = s.\ sig\left[e_p(nT_s)\right] \quad \text{—— (3)}$$

$$u(nT_s) = \hat{x}(nT_s) + e_q(nT_s)$$

$\hat{x}(nT_s)$ = the previous value of the delay ckt

$e_q(nT_s)$ = quantizer o/p = $V(nT_s)$

$$u(nT_s) = \left(u[n-1]T_s\right) + V(nT_s) \quad \text{—— (4)}$$

(5)

The present i/p of the delay unit

= previous o/p of the delay unit + the present quantizer o/p.

Assuming zero condn of Accumulator

$$u(nT_s) = \delta \sum_{j=1}^{n} sg[e_p(jT_s)]$$

Accumulated version of DM o/p $= \sum_{j=1}^{n} v(jT_s)$ — (5)

$$\hat{x}(nT_s) = (u[n-1]T_s)$$

$$= \sum_{j=1}^{n-1} v(jT_s) \quad \text{—— (6)}$$

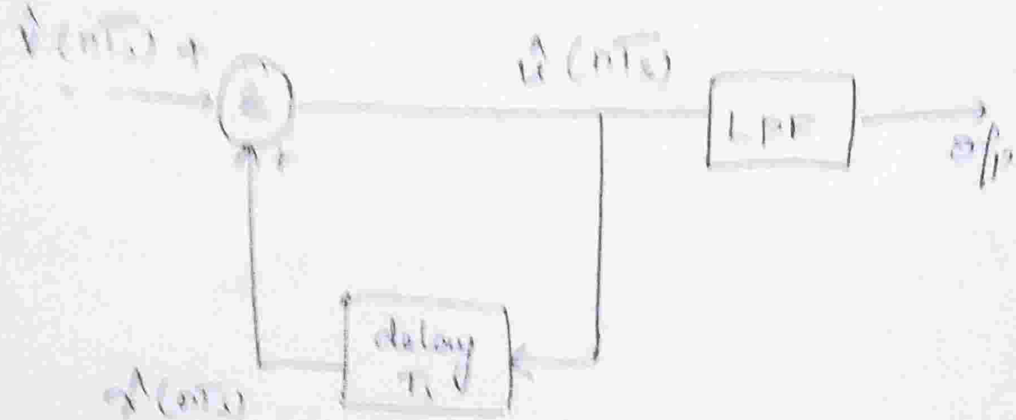
Delay unit o/p is an Accumulator o/p lagging by one sample.

from (5) & (6)

To find the structure of demodulator.

→ A staircase Approximation waveform will be the o/p of delta modulator which the step size is Δ (delta).

Delta demodulator:



→ The delta demodulator comprises of a low pass filter, summer, and delay ckt. The predictor ckt is eliminated here and hence no assumed input is given to the demodulator.

$\hat{v}(nT_s)$ → i/p to sample

$\hat{u}(nT_s)$ → summer o/p

$\hat{x}(nT_s)$ → delayed o/p.

→ A binary sequence will be given as an i/p to the demodulator.

→ A stair-case approximated o/p is given to the LPF.

→ LPF is used for many reasons, but the main reason is noise elemination for out of band signals.

→ The step size error that may occur at the transmitter is called granular Noise

Advantages of DM over DPCM.

* 1 - bit quantizer
* easy design of modulator and demodulator

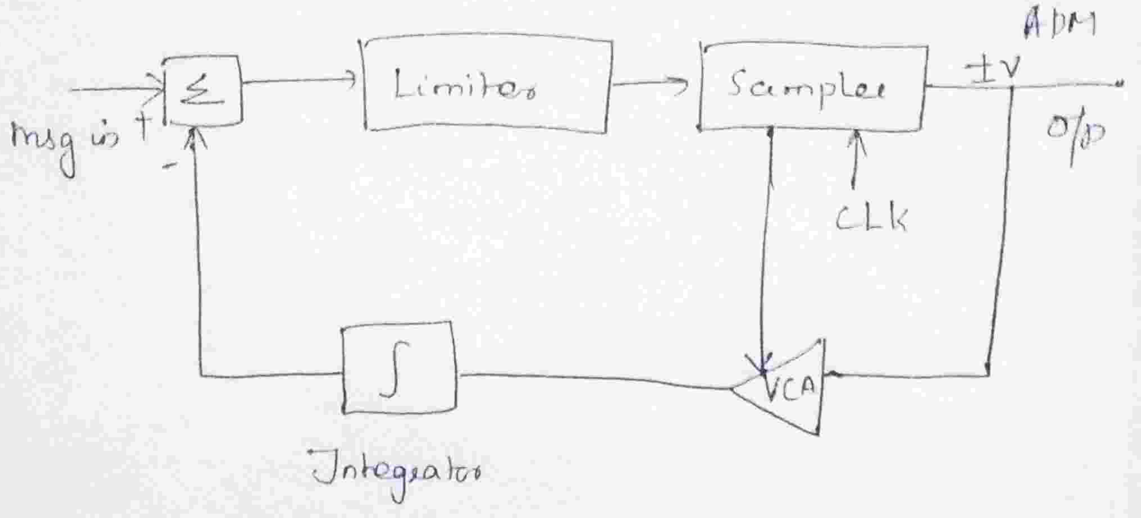
Dis Advantage:

Slope over load distortion (when Δ is small)

Granular noise (when Δ is large)

Adaptive Delta Modulation (ADM)

→ In digital Modulation, Certain problem of determining the step size, which influences the quality of the o/p wave.



→ A larger step size is needed to in the steep slop of modulating signal and a smaller step size is needed where the message has a small slope.

→ The minute details get missed in the process.

→ It would be better if control the adjustment of step size. according to requirement in order to obtain the sampling in the desired model.

→ This is the cocept of <u>Adaptive delta Modulation</u>

→ The gain of the voltage controlled Amplifier is adjusted by the o/p signal from the sampler. The Amplifier gain determines the step size and both are proportional.

→ ADM quantizes the difference b/w the value of the current sample and the predicted value of the next sample.

→ It uses a variable step height to predict the next values for the faithful reproduction of the fast varying values.

Linear Predictive Coding:

Linear predictive coding LPC is a tool which represents digital speech signal in linear predictive model.

→ This is mostly used for is audio signal processing, speech synthesis, speech recognition etc.

→ Linear prediction is based on the idea that the current sample is based on the linear combination of past samples.

Linear Prediction:

It is a mathematical operation where future value of a discrete time signal are estimated as a linear function of previous samples. In digital signal processing linear prediction is often called linear predictive coding and can thus be viewed as a subset of filter theory.

Filter design:

→ Filter design process is the process of designing a signal processing filter that satisfies a set of requirements, some of which are contradictory. The purpose to find a realization of the filter that meets each of the requirement to a sufficient degree to make it useful.

→ filter design process can be described as an optimization problem where each requirement contributes to an error function which should be minimized.

optimization:

optimization is the selection of a best element (with regard to some criteria) from some set of available alternatives.

LPC starts with the assumption that a speech signal is produced by a buzzer at the end of tube with occasional added hissing and popping sounds.

→ Although apparently crude, this model is actually a close approximation of the reality of speech production.

→ LPC analyses the speech signal by estimating the formats, removing their effects from the speech signal, and estimating the intensity and frequency of the remaining buzz.

→ The process of removing the formants is called inverse filtering, and the remaining signal after the subtraction of the filtered modelled signal is called the residue.

→ The number which describe the intensity and frequency of the buzz, the formats, and the residue signal can be stored or transmitted somewhere else. LPC synthesizes the speech signal by reversing the process: Use the ~~...~~ buzz parameters and the residue to create a source signal, Use the formants to create a filter

Because speech signal vary with time, this process is done on short chunks of the speech signal, which are called frames. Generally 30 to 50 frames per second.

→ It is one of the most powerful speech analysis techniques, and one of the most useful methods for encoding good quality speech at a low bit rate and provide extremely accurate estimate of speech parameters.

Properties of Line codes:

→ As the coding is done to make more bits transmit on a single signal, the bandwidth used is much reduced.

→ For a given bandwidth, the power is efficiently used.

→ The probability of error is much reduced.

→ error detection is done and the Bipolar too has a correction capability.

→ Power density is much favorable

→ The timing content is adequate

→ Long String of 1's and 0's is avoided to maintain transparency.

Types of Line coding:

- Unipolar
- polar
- Bi-polar.

Unipolar Signalling

→ Unipolar signalling is also called as ON-OFF keying or simply OOK.

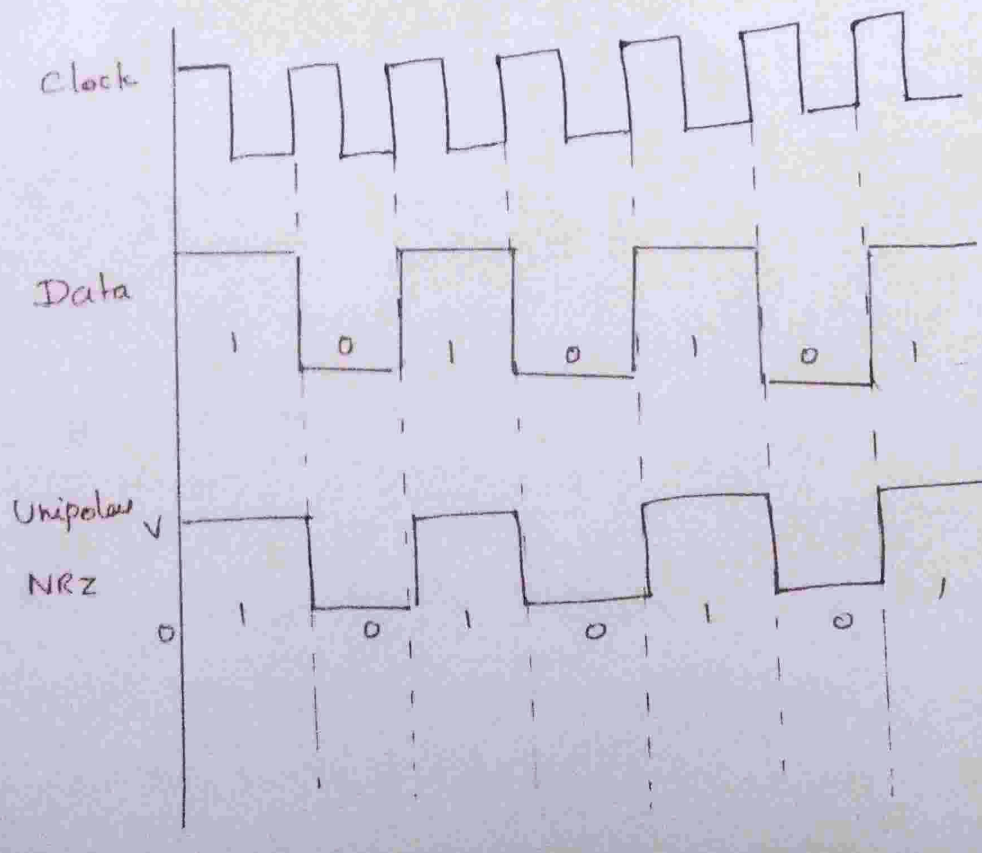
→ The presence of pulse represents a '1' and absence of pulse represents a '0'

Non Return to Zero - NRZ

Return to Zero. - RZ

Unipolar Non Return to Zero - NRZ.

→ In this type of unipolar signalling, a High in data is represented by a positive pulse called as Mark, which has a duration To equal to the symbol bit duration. A Low in data input has no pulse.



Advantages

* It is simple
* Lesser Bandwidth is required.

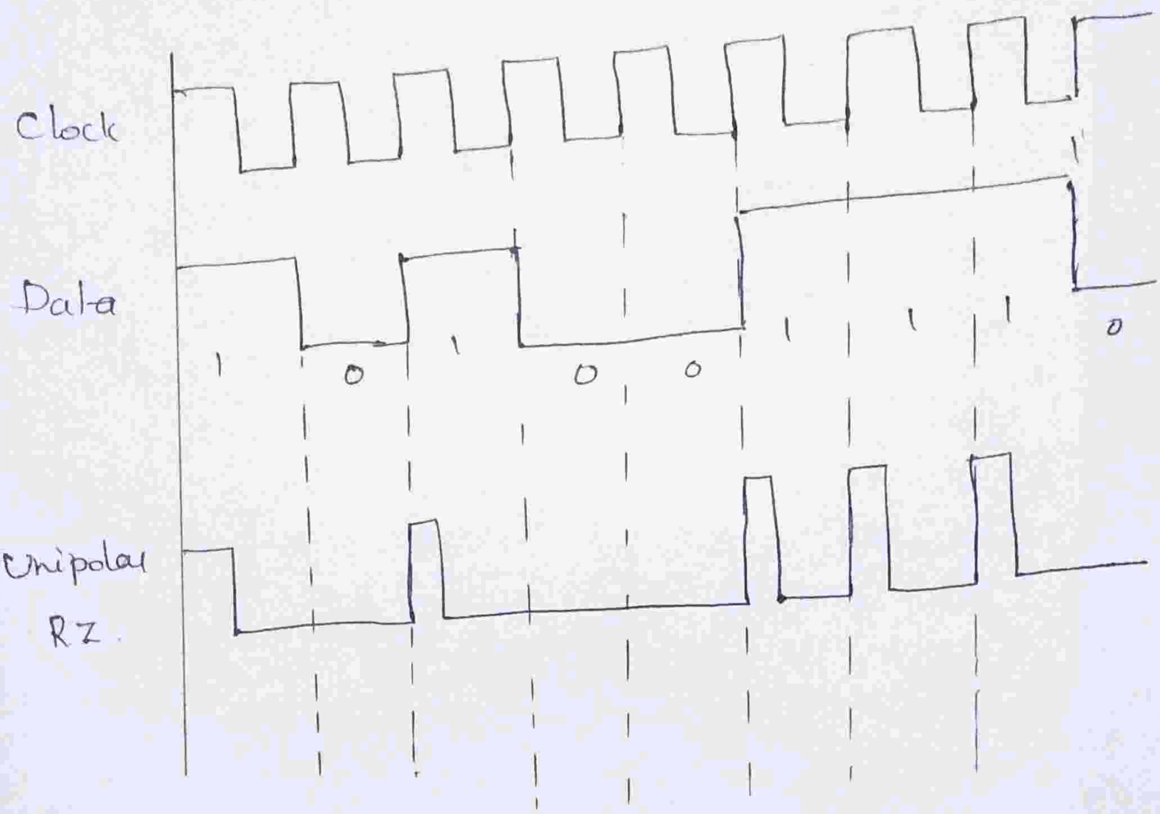
DisAdvantages.

* No error correction done
* Presence of low frequency components may cause the signal droop
* No clock is present
* Loss of synchronization is likely to occur

UNIPOLAR Return to Zero (RZ)

In this type of Unipolar signalling, a High in data, through represented by Mark Pulse, its duration 'To' is less than the symbol bit duration.

→ Half of the bit duration remains high but it immediately returns to zero and shows the absence of pulse during the remaining half of the bit duration.



Advantages:

It is simple

The spectral line present at the symbol rate can be used as a clock.

DisAdvantages:

* No error correction
* Occupies twice the bandwidth as unipolar NRZ.
* The signal droop is caused at the places where signal is Non-zero at 0 Hz.

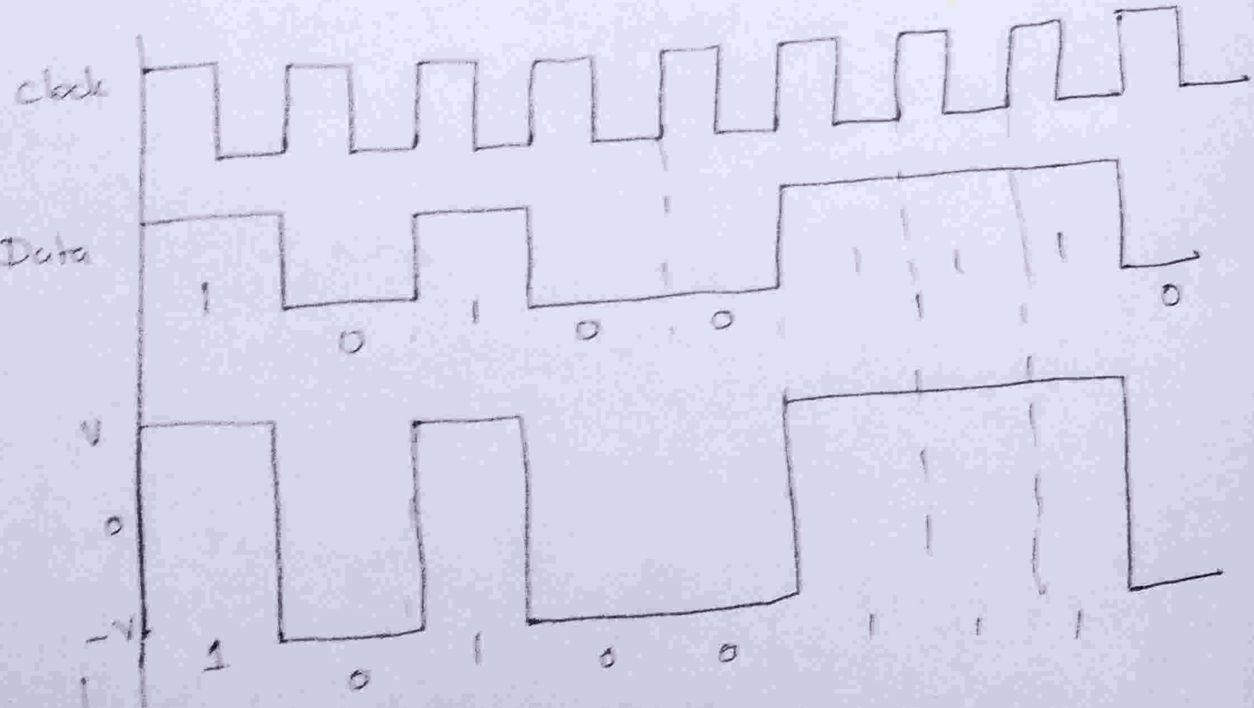
Polar Signalling

└ Polar NRZ

└ polar RZ

Polar NRZ

In this type of polar signalling, a High in data is represented by a positive pulse while a Low in data is represented by negative pulse.



Polar NRZ.

Advantages

It is simple

No low frequency components are present

DisAdvantages

* No error correction

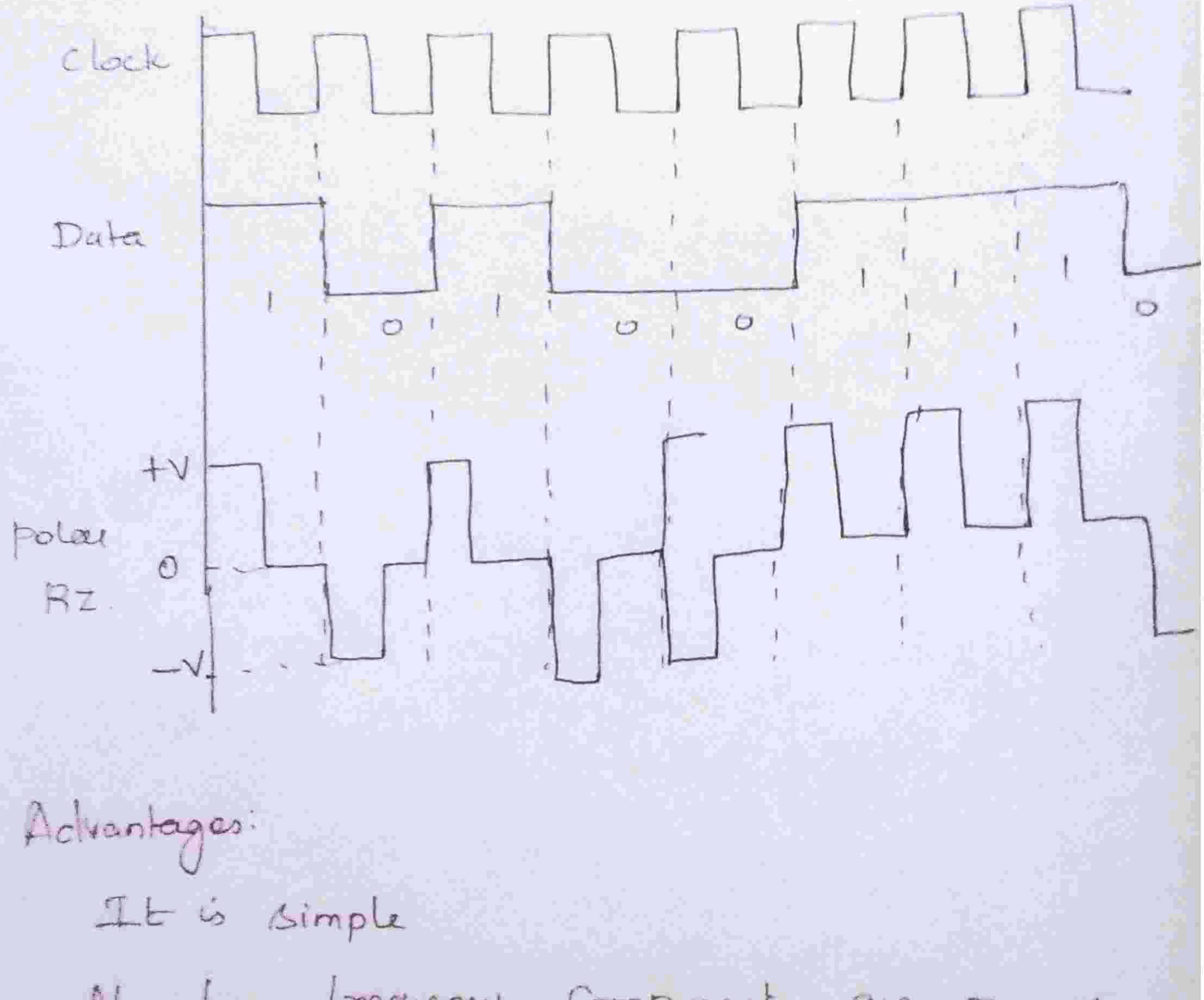
No clock is present

The signal droop is caused at the place when the signal is Non zero at 0 Hz.

Polar RZ

In polar RZ - a High in data, though represented by a Mark pulse, its duration T_0 is less than the symbol bit duration. Half of the bit duration remains high but it immediately returns to zero, and shows the absence of pulse during the remaining half of the bit duration.

However, for Low i/p, a negative pulse represents the data, and the zero level remains same for the other half of the bit duration.



Advantages:

It is simple

No Low frequency Components are present.

DisAdvantages:

* No error Correction

No clock is present

Occupies twice the bandwidth of polar NRZ.

The signal droop is caused at places

Bipolar signalling:

This is an encoding technique which has 3 voltages levels namely '+', '-' and '0'

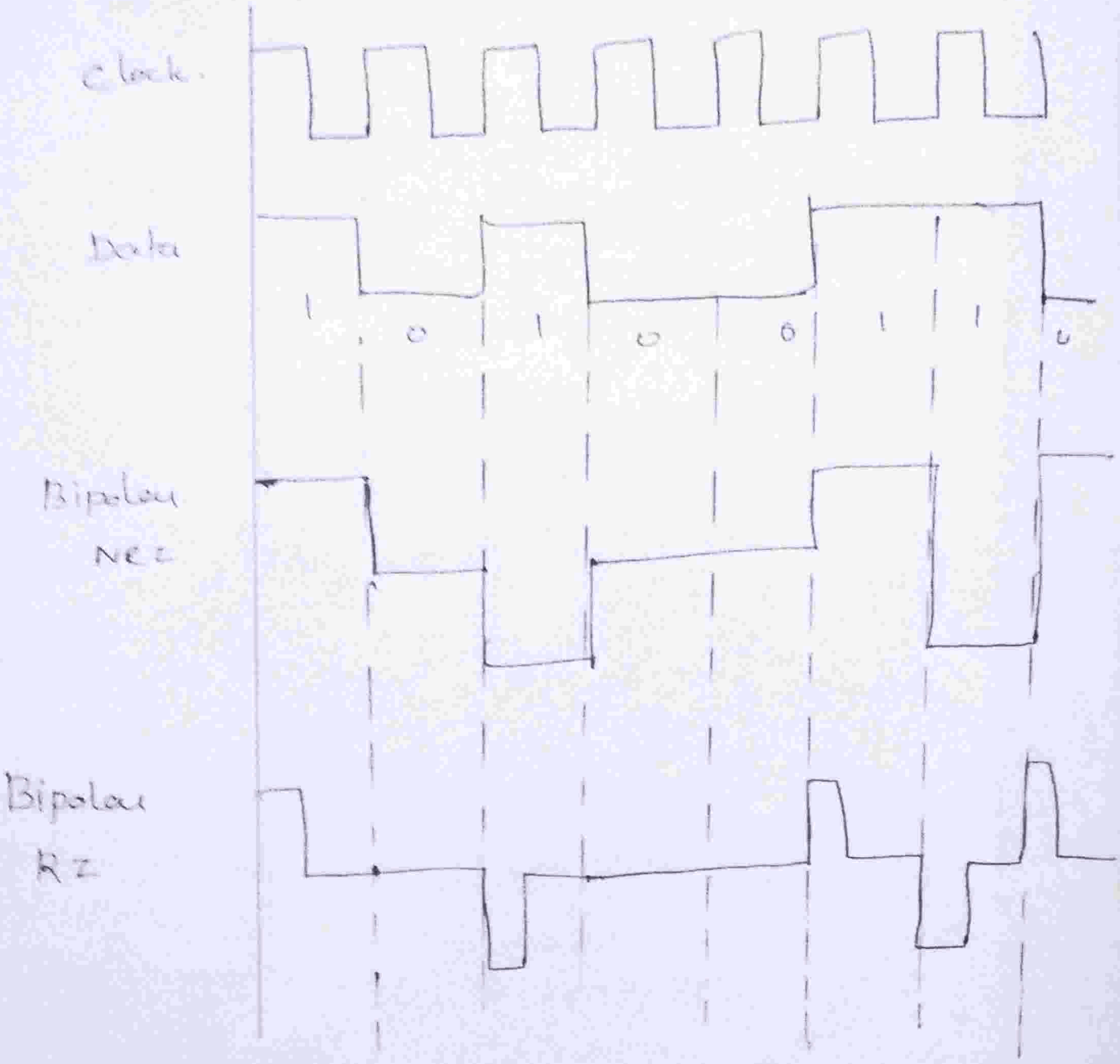
Alternate Mark inversion: (AMI)

For a 1, the vtg level gets a transition from '+' to '-' or from '-' to '+', having alternate 1's to be of equal polarity.

A '0' will have a zero voltage level.

└ Bipolar NRZ

└→ Bipolar RZ.



Advantages:

No low freq. components present

occupies lower bandwidth than ~~upper~~ unipolar and polar NRZ schemes.

→ Used for AC coupled lines.

→ error detection is present.

Dis Advantages

No clock is present

Long string of data causes loss of synchronization

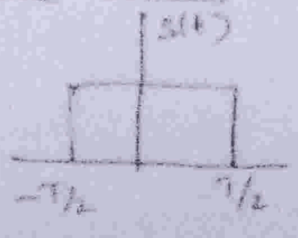
Power spectral Density (PSD)

The function which describes how the power of a signal got distributed at various frequencies in the freq. domain is called Power Spectral Density (PSD).

→ PSD is the fourier Transform of Auto correlation

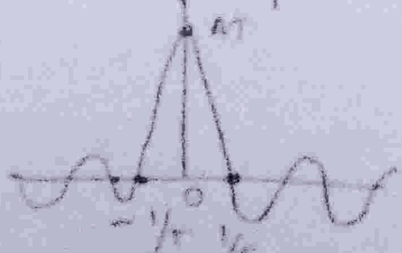
→ It is the form of a rectangular pulse.

Time domain



$$s(t) = \begin{cases} A, & |t| < T/2 \\ 0, & |t| > T/2 \end{cases}$$

freq domain



$$S(f) = AT \frac{\sin(\pi f t)}{\pi f t}$$

Manchester

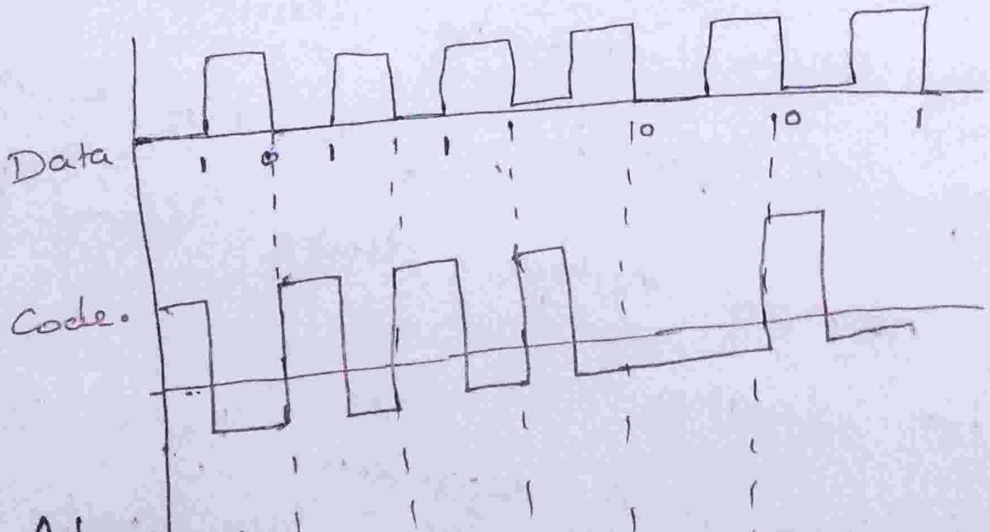
In telecommunication and data storage ~~Manch~~ Manchester Code is a line code in which the encoding of each data bit is either low then High, or high then low, for equal time.

In this Coding, a binary 1 is represented by a pulse that has positive voltage during the first half of the bit duration and negative voltage during second half of the bit duration.

A binary 0 is represented by a pulse that is negative during the first half of the bit duration and positive during the second half of the bit duration. The negative or positive mid bit transmission indicates a binary 1 or binary 0 respectively.

→ The Manchester code is classified as an instantaneous transition code. it has no memory.

→ The code is also called diphase because a square wave with a 0° phase is used to represent a binary 1 and a square wave with a phase of 180° used to represent a binary 0; vice versa.



Advantage:

Good error, per rate, performance. identical to polar NRZ.

* This code include a zero dc content on an individual pulse basis, so no pattern of bits can cause dc build up.

* Mid bit transitions are always present making it is easy to extract timing Information.

Features:

Manchester format does not have DC Component but provides proper clocking.

DIS Advantages:

* A Larger bandwidth than any of the other common codes.

* It has no error detection capability and hence, performance monitoring is not possible.

UNIT: III

BASEBAND TRANSMISSION & RECEPTION:

Introduction:

The digital Communication, the digital signal Used is baseband transmission occupies the entire bandwidth of the network media to transmit a single data signal. Baseband Communication is bidirectional, allowing Computers to both send and receive data using a single cable.

Baseband Modulation:

Modulation is a process of an information bearing signal must be Conform to the limits of its channel.

→ Modulation Can be performed is a two step process.

1. Baseband → Shaping the spectrum of i/p bits to fit in a limited spectrum
2. passband → Modulating the baseband sgl to the s/m radio frequency carrier.

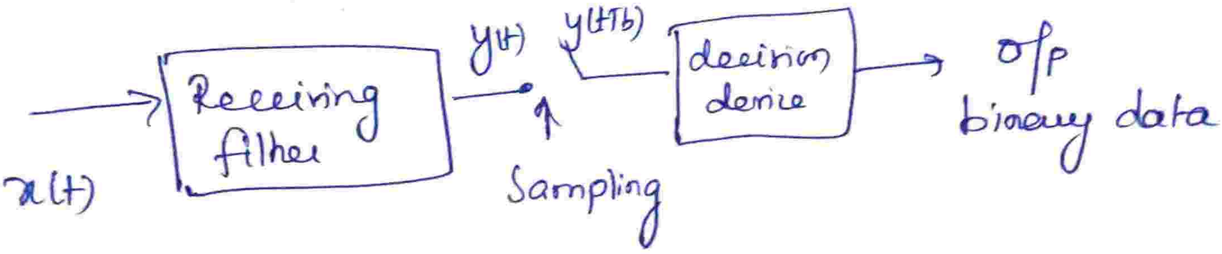
Need for baseband Modulation:

→ An analog signal has a finite B.W

→ A digital stream or sgl, with sharp transition has an infinite B.W

Inter Symbol Interference

Consider the Receiving Section:

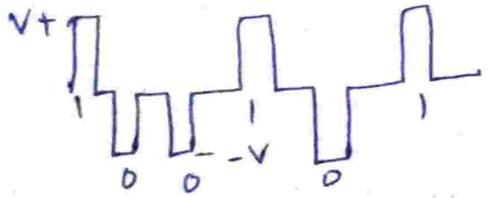


Receiver PAM transmission

$$x(t) = \sum_{k=-\infty}^{\infty} A_k \, g(t - kT_b) \quad \text{—— (1)}$$

PAM → Best power & B.W efficiency.

i/p → 1 0 0 1 0 1



ON time is very less, so Consider the impulse signal.

(2)

$$a(t) = \sum_{k=-\infty}^{\infty} a_k\, \delta(t - kT_b) \quad \text{—(1)} \qquad a_k < \begin{matrix} -V, & b_k = 0 \\ +V & b_k = 1 \end{matrix}$$

$$+V\delta(t+T_b) - V\delta(t) + V\delta(t - T_b) - V\delta(t - 2T_b) + V\delta(t - 3T_b)$$

$$s(t) = a(t) * g(t)$$

↳ Transmit filter

$$= \sum_{k=-\infty}^{\infty} a_k\, \delta(t - kT_b) * g(t)$$

$$s(t) = \sum_{k} a_k\, g(t - kT_b) \quad \text{—— (2)}$$

$$s(t) = \sum_{k=-\infty}^{\infty} a_k\, g(t - kT_b) \quad \text{—— (2)}$$

Channel $\rightarrow s(t) * h(t) + w(t)$

$$y'(t) \Rightarrow \sum_{k=-\infty}^{\infty} a_k\, g(t - kT_b) * h(t) + w(t)$$

$$y(t) = y'(t) * x(t)$$

$$\boxed{y(t) = \sum a_k\, g(t - kT_b) * h(t) + w(t)}$$

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k\, \underbrace{p(t - kT_b)}_{\text{↳ Combined impulse.}} + n(t)$$

↳ noise

μ ↓ scaling factor

$t = t_i$

$$y(t_i) = \mu \sum_{k=-\alpha}^{\alpha} a_k (p\,t_i - kT_b) + n(t_i)$$

$t_i = iT_b$, $k = i$

$$y(t_i) = \sum_{k=-\alpha}^{\alpha} a_k (p\,kT_b - iT_b) + \mu \sum_{\substack{k=-\alpha \\ k \neq i}}^{\alpha} a_k (p\,iT_b - kT_b) + n(t_i)$$

$\Downarrow$

$p(0) = 1$

$$y(t_i) = \underbrace{\mu a_i}_{i^{th} \text{ transmitted bit}} + \underbrace{\mu \sum_{\substack{k=-\alpha \\ k \neq i}}^{\alpha} a_k \left(p(iT_b - kT_b)\right) + n(t_i)}_{\text{ISI}}$$

$y(t_i) \longrightarrow$ Applied to decision device.

ISI $\rightarrow$ It is the process of effect of otherbits interference with o/p of required bit.

Elimination of ISI

→ proper design of pulse spectrum.

→ Individual spectrum of the pulse should be separated by a bit period (T_b)

Nyquist Criterion for distortionless Transmission

$$y(t_i) = \mu a_i + \mu \sum a_k p((i-k)T_b) + n(t_i)$$

According to Nyquist Condn

$$P[(i-k)T_b] = \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases} \quad \begin{matrix} i-k=0 \\ n=0. \end{matrix}$$

$i-k = n$,

$$p \begin{cases} p(nT_b) = 1 & : n=0 \\ 0 & \text{otherwise} \end{cases}$$

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_b)$$

$$S(f) = f_b \sum_{n=-\infty}^{\infty} \delta(f - nT_b)$$

$$\frac{1}{T_b} \sum_{n} \delta\left(f - \frac{n}{T_b}\right)$$

$$R_b \sum_{n} \delta(f - nR_b)$$

$$p_s(t) = p(t) \sum_{n=-\infty}^{\infty} \delta(t - kT_b)$$

$$p_s(t) = \sum_{n=-\infty}^{\infty} p(nT_b)\, \delta(t - nT_b)$$

$$P_s(f) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} p(nT_b)\, \delta(t - nT_b)\, e^{-j2\pi f t}\, dt$$

$$P_s(f) = \int_{-\infty}^{\infty} \sum_{n} p(nT_b)\, \delta(t - nT_b)\, e^{-j2\pi f t}\, dt$$

$$P_s(f) = 1, \quad n = 0$$

$$P_s(f) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi f t}\, dt$$

$$\int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi f t}\, dt = 1$$

$$\boxed{P_s(f) = 1}$$

$$P_s(f) = P(f) * S(f)$$

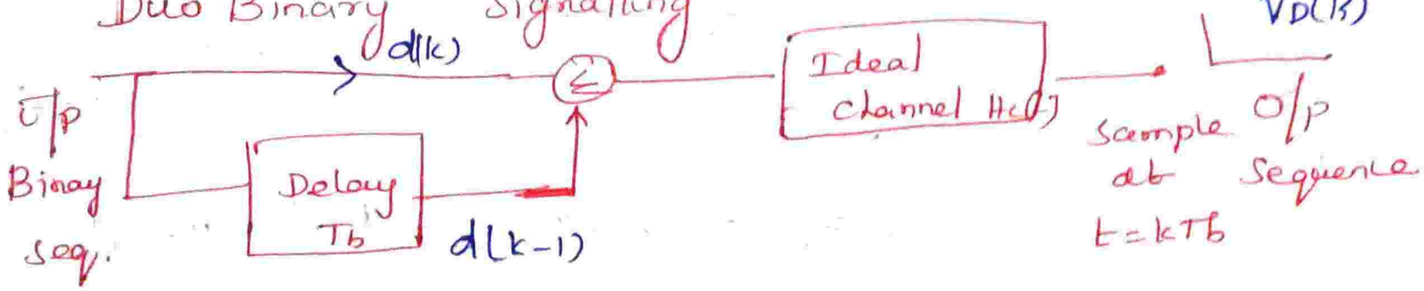
$$1 \Rightarrow P(f) * R_b \sum_{n} \delta(t - R_b)$$

$$\boxed{1 = P(f) * R_b \sum_{n} \delta(t - kR_b)} \qquad R_b = 1/T_b.$$

Correlative Coding:

Intersymbol interference is an undesirable phenomenon that produces degradation in s/m performance. But by adding intersymbol interference to the transmitted signal in a controlled manner, it is possible to achieve a bit rate of $2B_0$ bits per second in a channel of B.W B_0 hertz. Such schemes are called correlative coding or partial response signaling schemes.

Duo Binary Signalling



Duo implies doubling of the transmission capacity of a straight binary symbol.

$V_D(k) = d(k) + d(k-1)$

Transfer fun.

delay element $\Big\}$ $H_D(f) = e^{-j2\pi f T_b}$

$1 + H_D(f) = 1 + e^{-j2\pi f T_b}$

Cascaded overall transfer fun.

$$H(f) = [1 + H_D(f)]\, H_c(f)$$

$$= \left[1 + e^{-j2\pi f T_b}\right] H_c(f)$$

$$= \left[e^{-j\pi f T_b} \cdot e^{j\pi f T_b} + e^{-j\pi f T_b} \cdot e^{-j\pi f T_b}\right] H_c(f)$$

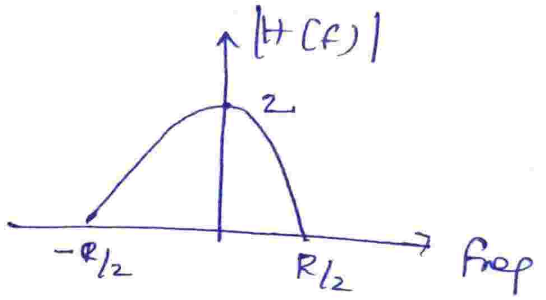
$e^{j\omega} + e^{-j\omega} = 2\cos\omega.$

$$= e^{-j\pi f T_b}\left[e^{j\pi f T_b} + e^{-j\pi f T_b}\right] H_c(f)$$

$$\boxed{H(f) = e^{-j\pi f T_b} \cdot 2\cos \pi f T_b \cdot H_c(f)}$$

$\Rightarrow$

Pulse Shape



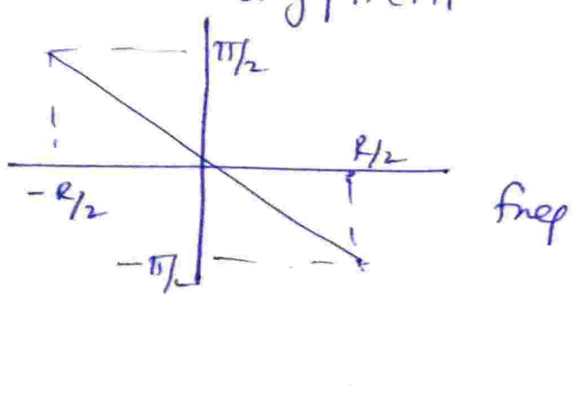
Amp. Response.

B.W = B ⇒ 1/2 R

$$T.F = H_c(f) = \begin{cases} 1, & \text{for } |f| \le R/2 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} 2e^{-j2\pi f T_b} \cos \pi f T_b & \text{for } |f| \le R/2 \\ 0 & \text{elsewhere.} \end{cases}$$

Phase Response



Impulse Response

$$h(t) = IFT\left[H(f)\right]$$

$$= IFT\left[2e^{-j2\pi f T_b}\cos(\pi f T_b) \cdot H_c(f)\right]$$

$$|f| \leq R_b/2$$

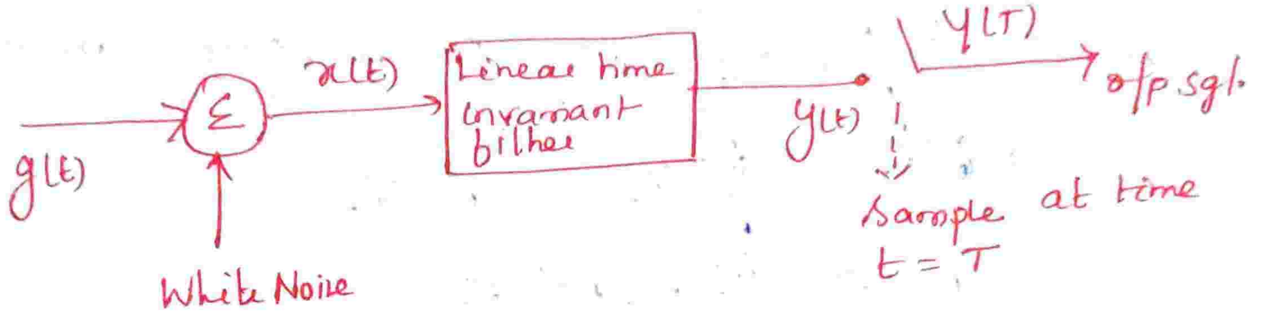
$$h(t) = IFT\left[2e^{-j\pi f T_b}\cos(\pi f T_b)\right].$$

$$\boxed{h(t) = \frac{\sin \pi t/T_b}{\pi T/T_b} + \sin\frac{\pi(t-T_b)/T_b}{\pi(t-T_b)/T_b}}$$

$$\boxed{h(t) \Rightarrow \frac{T_b^2 \sin(\pi t/T_b)}{\pi t\,(T_b - t)}}$$

Matched filter – Performance Analysis

Consider the Receiver Model, involving a linear time invariant filter of impulse response $h(t)$.

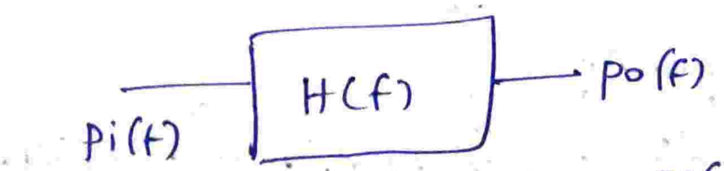


The filter i/p $x(t)$ consists of a pulse signal $g(t)$ corrupted by additive channel noise $w(t)$ is

$$SNR = \gamma^2 = \frac{p^2(t)}{\sigma^2}$$

$p^2(t) \rightarrow$ power

$p(t) = x_{01} - x_{02}$



$P_o(f) = H(f) \cdot P_i(f)$

$$p_o(t) = \int_{-\infty}^{\infty} P_o(f)\, e^{j2\pi ft}\, df$$

$$p_o(t) = \int_{-\infty}^{\infty} H(f)\, P_i(f)\, e^{j2\pi ft}\, df$$

$$p_o^2(t) = \left|\int_{-\infty}^{\infty} H(f)\, P_i(f)\, e^{j2\pi ft}\, df\right|^2 \quad \text{——} \; (1)$$

$\sigma^2 = E\{[n_o^2(t)]\} \longrightarrow$ power spectral density

$$n_i^2(f) \longrightarrow \boxed{H^2(f)} \longrightarrow n_o^2(f)$$

$$n_o(f) = H^2(f) \cdot n_i^2(f)$$

$$N_i(f) = \frac{N_o}{2}$$

$$n_o(f) = H^2(f) \cdot \frac{N_o}{2}$$

$$\sigma^2 = \int_{-\infty}^{\infty} \frac{N_o}{2} H^2(f)\, df \quad \text{——} \; (2)$$

$$\gamma^2 = \frac{p_o^2(t)}{\sigma^2} = \frac{\left|\int_{-\infty}^{\infty} H(f)\, P_i(f)\, e^{j2\pi ft}\, df\right|^2}{\frac{N_o}{2}\int_{-\infty}^{\infty} H^2(f)\, df}$$

Schwarz's inequality

$$\left|\int_{-\infty}^{\infty} x(f)\, y(f)\, df\right|^2 \le \int_{-\infty}^{\infty} x^2(f)\, df \cdot \int_{-\infty}^{\infty} y^2(f)\, df$$

$$x(f) = H(f)$$
$$y(f) = P_i(f)\, e^{j2\pi ft}$$

$$\left| \int_{-\alpha}^{\alpha} H(f)\, P_i(f) e^{j2\pi ft} \right| \leq \int_{-\alpha}^{\alpha} H_i(f)\, df \cdot \int_{-\alpha}^{\alpha} P_i(f) e^{j2\pi ft}\, df$$

$$\frac{\left| \int_{-\alpha}^{\alpha} H(f)\, P_i(f) e^{j2\pi ft} \right|}{\frac{N_0}{2} \int_{-\alpha}^{\alpha} H^2(f)\, df} \leq \frac{\int_{-\alpha}^{\alpha} P_i^2(f) \cdot df}{\frac{N_0}{2}}$$

$$\gamma^2_{max} \leq \frac{2}{N_0} \int_{-\alpha}^{\alpha} P_i^2(f)\, df \Longrightarrow \text{SNR} \rightarrow \text{of}$$

Matched filter.

$$X(f) = k\, y^*(f)$$

$$\underbrace{H(f) = k\, P^*(f)}\, e^{-j2\pi ft}$$

$\longrightarrow$ Transfer fun in frequency domain.

$$\boxed{H(t) = \int_{-\alpha}^{\alpha} H(f)\, e^{j2\pi ft}\, df}$$

$$\boxed{\gamma^2 = \frac{2}{N_0} \int_{-\alpha}^{\alpha} P_i^2(f)\, df} \quad \text{SNR}$$

SNR depends upon the i/p sgl only.

$$\frac{S}{N}\Big|_{max} = \gamma^2_{max} = \int_{-\infty}^{\infty} \frac{X(f)^2}{S_{ni}(f)}\, df$$

$$S_{ni}(f) = \frac{N_0}{2}$$

$$\gamma^2 = \int_{-\infty}^{\infty} \frac{X(f)^2}{N_0/2}\, df$$

$$= \frac{2}{N_0} \int_{-\infty}^{\infty} X(f)\, df$$

Parsevals theorem

$$\int_{-\infty}^{\infty} X^2(f)\, df = \int_{-\infty}^{\infty} x^2(t)\, dt = E$$

$$\gamma^2_{max} = \frac{2 \times E}{N_0} = \frac{E}{N_0/2} = \frac{\text{Energy of sgl}}{\text{powerspectral density of additive noise}}$$

Transfor fun:

$$H(f) = k^* \frac{X^*(f)}{S_{ni}(f)} e^{-j2\pi f t}$$

$$X_o(f) = H(f) \cdot X_i(f)$$

$$X_o(f) = H(f) \cdot X_i(f)$$

$$X_o(f) = k\frac{X^*(f)}{S_{nr}(f)} e^{-j2\pi fT} \cdot x(f)$$

$X^*(f) \rightarrow$ conjugate of $x(f)$

$$X^*(f) \cdot x(f) = |x^2(f)|$$

$$X_o(f) = \frac{k}{N_o/2} x^2(f) \cdot e^{-j2\pi fT}$$

$$X_o(t) = \int_{-\infty}^{\infty} X_o(f) \cdot e^{j2\pi ft} df$$

$$X_o(t) = \int_{-\infty}^{\infty} \frac{k}{N_o/2} x^2(f) e^{-j2\pi fT} \cdot e^{j2\pi ft} df$$

$$\frac{2k}{N_o} \int_{-\infty}^{\infty} x^2(f) e^{j2\pi f(t-T)} df$$

$t = T$

$$X_o(t) = \frac{2k}{N_o} \int_{-\infty}^{\infty} x^2(f) e^{j2\pi f(T-T)} df$$

$$X_o(t) = \frac{2k}{N_o} \int_{-\infty}^{\infty} x^2(f) \cdot df$$

$$\frac{2k}{N_o} \int_{-\infty}^{\infty} x^2(f) df$$

$$\frac{2k}{N_o} \int_{-\infty}^{\infty} x^2(t) dt$$

$$\Rightarrow \frac{2k}{N_o} \cdot E$$

Normalization

$$\frac{2k}{N_0} \cdot E$$

$$\frac{2k}{N_0} = 1$$

$$\boxed{X_0(f) = E}$$

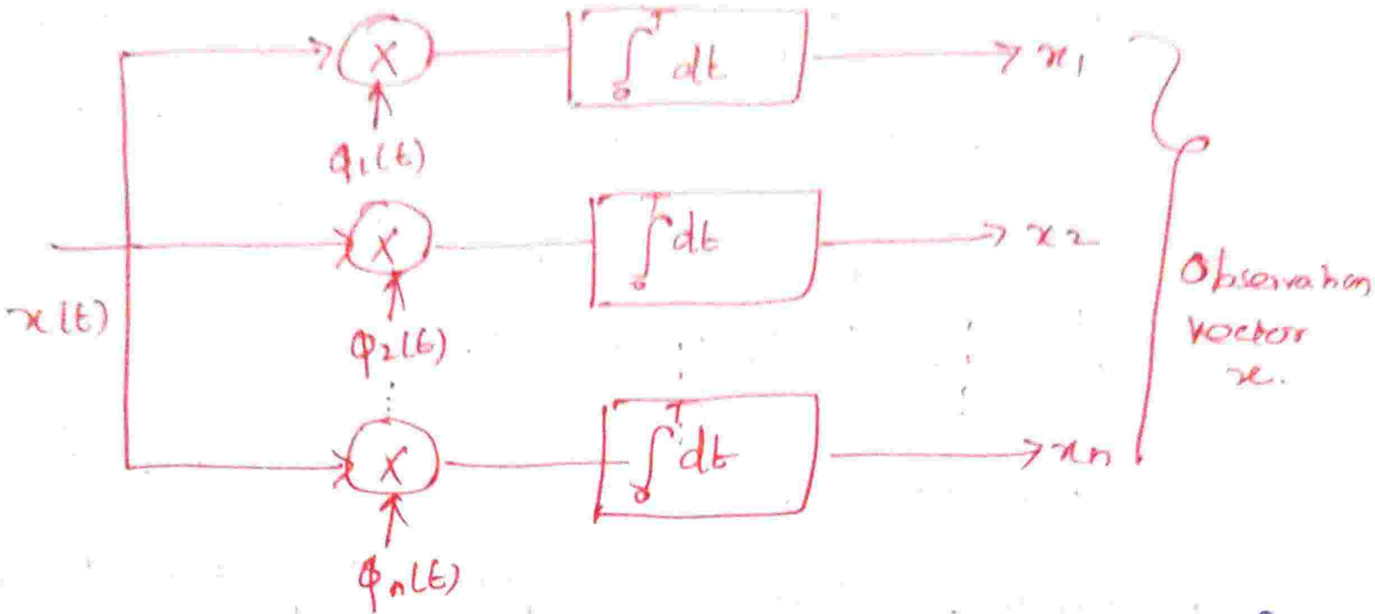
Correlation Receiver

→ The optimum receiver consists of two sub system.

1. detector or demodulator
2. signal decoder

→ Detector or demodulator consists of N-bank of correlator, each one having one orthogonal basis fun that are locally generated.

→ This bank of correlator operates on the received signal x(t) and produce the observation vector x.



→ This bank of correlation operates on the received signal $x(t)$, $0 \le t \le T$, to produce the observation vector x.

Signal Transmission Decoder:

→ Signal transmission decoder is implemented in the form of a maximum likelihood decoder that operate on the observation vector x to produce an estimate $\hat{m}$ of the transmitted symbol m_i, $i = 1, 2, \dots M$.

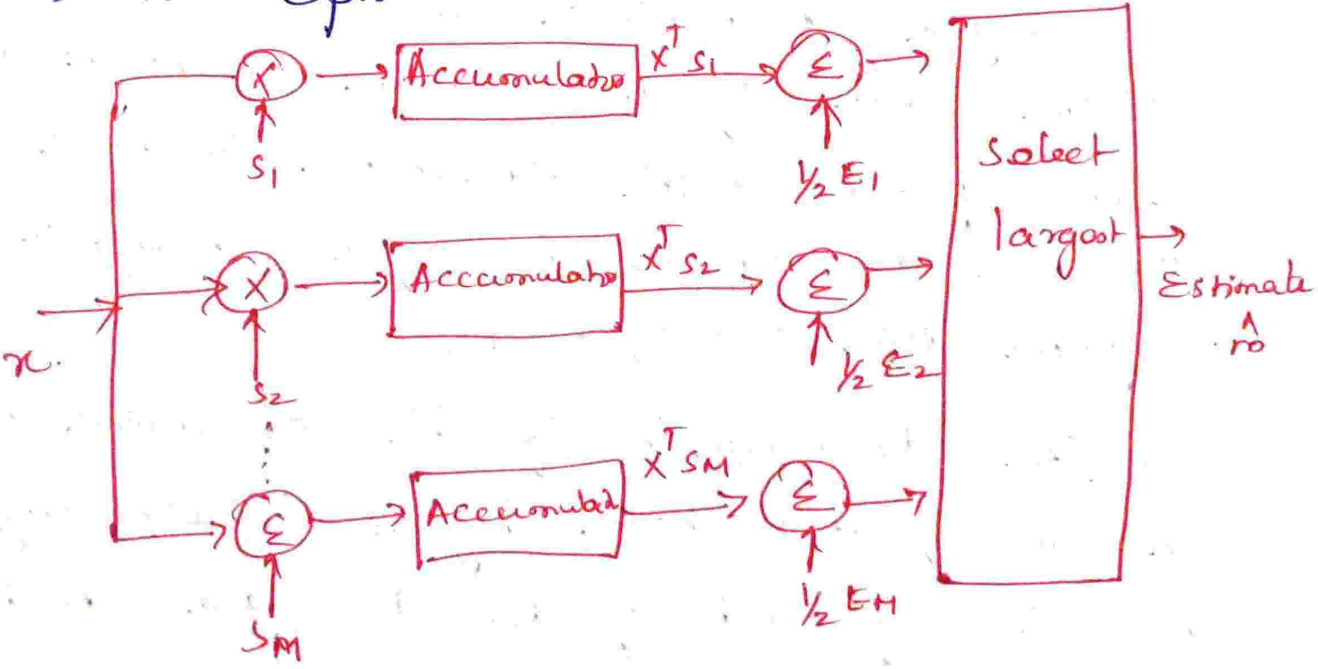
→ The N elements of the observation vector X are first multiplied by the corresponding 'N' elements of each of the M signal vectors $s_1, s_2, \dots s_M$.

→ The resulting products are successively summed in accumulator to form the corresponding set of inner products $\{x^T s_k\}$, $k = 1, 2, \ldots M$.

→ The inner products are corrected for the transmitted signal energies may be unequal.

→ Finally, the largest in the resulting set of number is selected and an appropriate decision on the transmitted message is made.

→ The optimum receiver



Adaptive Equalization

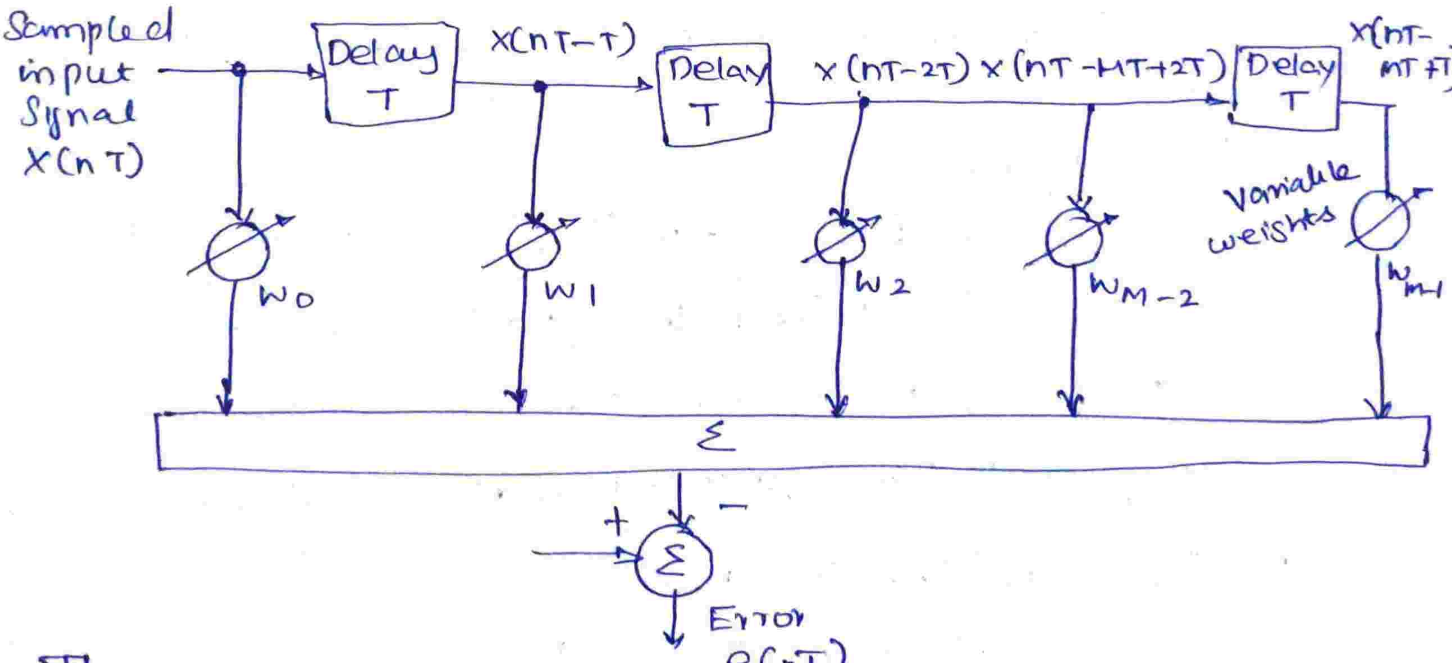
Most of the channels are made up of individual links. For eg, in the switched telephone network, the distortion induced depends upon

i) Transmission characteristics of individual links

ii) Number of links in connection.

Hence, the fixed pair of transmit and receive filters will not serve the equalization problem completely. The transmission characteristics of the channel keep on changing. Hence adaptive equalization is used.

Structure of adaptive equalizer.



The output $y(nT) = \sum_{i=0}^{M} w_i\, x(nT - iT)$

The weights w_i on the taps are basically adaptive filter coefficients. A know sequence $\{d(nT)\}$ is transmitted first. This sequence is known to the receiver.

The response sequence $y(nT)$ is observed, the error sequence between the two sequences is calculated i.e,

$$e(nT) = d(nT) - y(nT), \quad n = 0, 1 \cdots N-1.$$

If there is no distortion in the channel, then $d(nT)$ and $y(nT)$ will be exactly same producing zero error sequence. Then the weights of the filter i.e, W_i are changed recursively such that error $e(nT)$ is minimized.

operating modes:-

i) Training mode ii) Decision directed mode.

i) Training mode:

When the switch 'sw1' is in position 2, training sequence is applied to equalizer. The training sequence is basically maximal length Pseudo Noise (PN) sequence. It generates the synchronized version in the receiver.

ii) Decision directed mode.

$$e(nT) = b(nT) - y(nT)$$

Adaptive equalizer sets switch 'sw1' in position 1 after completion of training mode, An error signal is given above.

UNIT: IV

DIGITAL MODULATION SCHEME:

Geometric Representation of Signal:

Consider 'M' no. of energy signal,

$S_i(t) = \{ S_1(t), S_2(t), \dots S_m(t) \}$

in terms of 'N' no. of orthogonal function

$\phi_j(t) = \{ \phi_1(t), \phi_2(t), \dots \phi_n(t) \}$

Linear relation between $S_i(t)$ & $\phi_j(t)$ as

$$S_i(t) = S_{i1}\phi_1 + S_{i2}\phi_2 + S_{i3}\phi_3 + \dots \phi_N(t) S_{iN}$$

$$S_i(t) = \sum_{j=1}^{N} S_{ij}\, \phi_j(t) \quad \text{—— (1)}$$

$S_{ij} \rightarrow$ Co-efficient of expansion.

$$S_{ij} = \int_0^T g_i(t)\, \phi_j(t)\, dt \quad \text{—— (2)}$$

$T \rightarrow$ is the duration of symbol $S_i(t)$

$\phi_1(t), \phi_2(t) \dots \phi_N(t)$ are orthogonal.

Basis

$$\int_0^T \phi_i(t)\, \phi_j(t)\, dt = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

Example

Two dimentional signal with three Symbol

$M = 3, \quad s_1, s_2, s_3$

$N = 2, \quad \phi_1\ \phi_2$

$$S_i(t) = \sum_{j=1}^{2} s_{ij}(t) \cdot \phi_j(t) \; ; \; i = 1, 2, 3$$

$$S_1(t) = s_{11}(t)\, \phi_1(t) + s_{12}(t)\, \phi_2(t)$$

$$S_2(t) = s_{21}(t)\, \phi_1(t) + s_{22}(t)\, \phi_2(t)$$

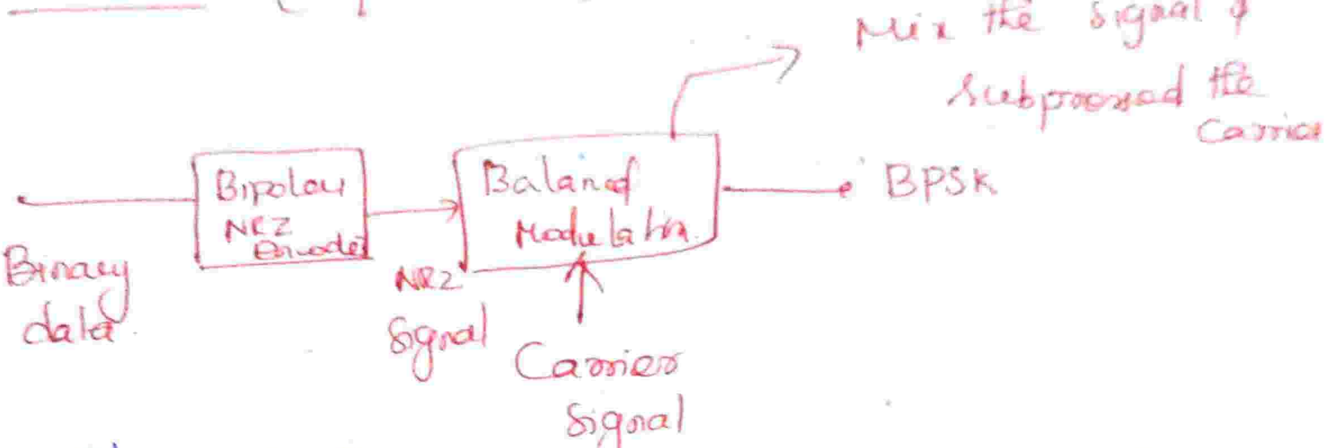
$$S_3(t) = s_{31}(t)\, \phi_1(t) + s_{32}(t)\, \phi_2(t)$$

$\phi_1, \phi_2 \rightarrow$ perpendicular to each other.

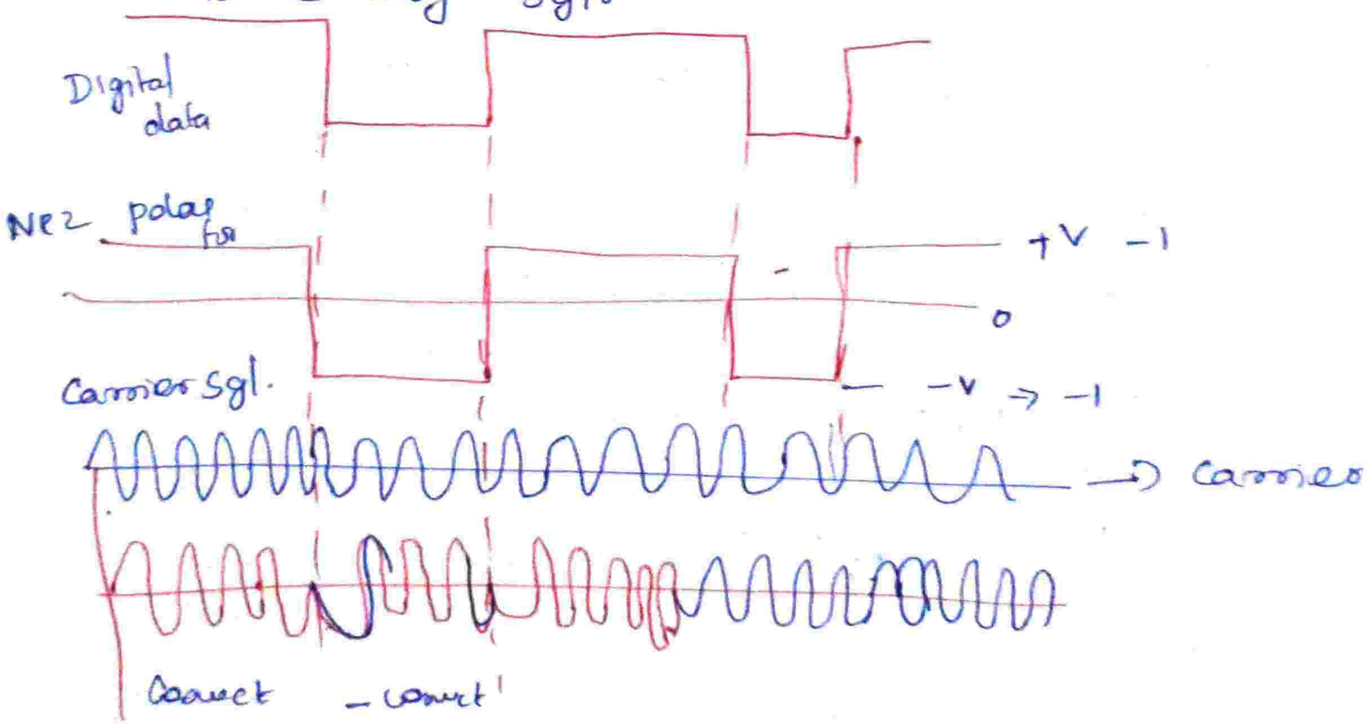
$\phi_1, \phi_2 \rightarrow$ Euclidance space.

Generation and detection of PSD and BER of Coherent BPSK.

BPSK (Generation)



→ Phase of the carrier signal is varied in accordance to the msg sgl.



BPSK signal → Binary signals '0' & '1'

Carrier signal

$s(t) = A\cos(2\pi f_0 t)$

A → peak value of sine carrier signal

Power dissipated

$P = \frac{1}{2}A^2 \Rightarrow A = \sqrt{2P}$.

Symbol 1:

$s(t) = \sqrt{2P}\,(\cos 2\pi f_0 t)$ —— (1)

Symbol '0'

There will be a phase shift of $180°$ (π - radians)

$s_2(t) = \sqrt{2P}\cos(2\pi f_0 t + \pi)$

$\cos(\phi + \pi) = -\cos\phi$

$s_2(t) = -\sqrt{2P}\cos(2\pi f_0 t)$ —— (2).

Using eq (1) & (2).

$s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t)$

$b(t) = +1 \Rightarrow$ symbol-'1'

$b(t) = -1 \Rightarrow$ symbol-'0'

$s(t) \Rightarrow b(t)\sqrt{2P}\cos(2\pi f_0 t) \longrightarrow (1)$

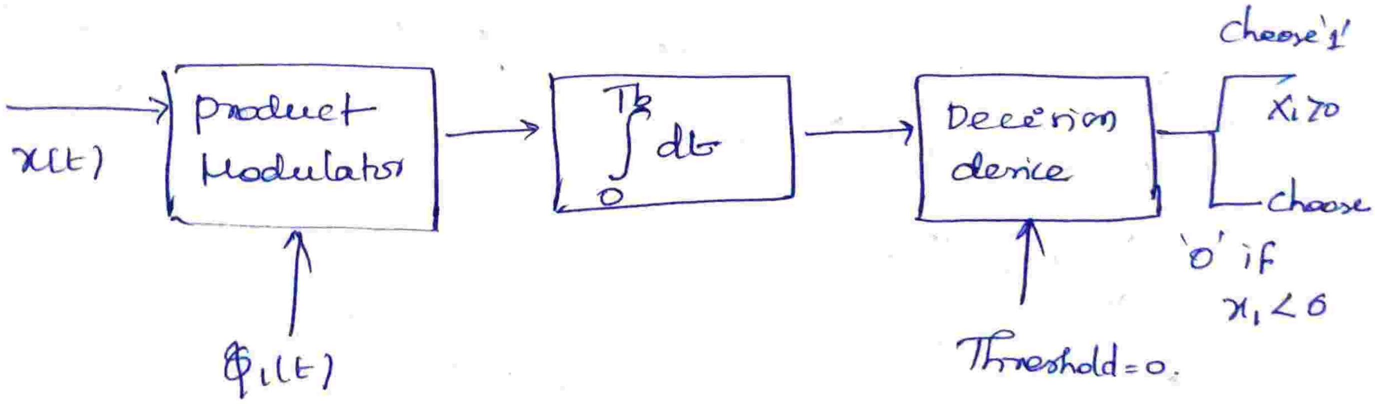
$\sqrt{2P_s}\cos\omega_0 t \longrightarrow (1)$ for '1' symbol

$\sqrt{2P_s}\cos(\omega_0 t + \pi) \longrightarrow (2)$ for '0' symbol

$\sqrt{2P_s} = \sqrt{2P_s} \times \dfrac{\cancel{\sqrt{T_p}}}{\sqrt{T_p}}$

phase changes θ = $s(t) = b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta)$ $\longrightarrow (2)$

Receiver



(i) Square law device. $(\cos 2\pi f_0 t + \theta)$

$\cos^2\omega = \dfrac{1+\cos 2\omega}{2} \Rightarrow = (\cos 2\pi f_0 t + \omega) \Rightarrow \dfrac{1+\cos 2(2\pi f_0 t + \omega)}{2}$

$\frac{1}{2} + \frac{1}{2}(\cos 2\pi f_0 t + \omega)$

$\hookrightarrow$ DC component

→ To detect the original binary sequence from noisy BPSK signal $x(t)$, the received signal from the channel is applied to the correlator.

→ The locally generated coherent reference signal $\phi_1(t)$, is also applied to the correlator. The o/p of the multiplier is integrated over one bit period T_b. The correlator o/p x_1 is compared with a threshold of zero volts.

→ If the correlator o/p is exceeded the threshold, the receiver decides in favour of symbol '1'.

→ If the correlator o/p is not exceeded the threshold, the receiver decides in favour of symbol '0'.

Bandwidth:

B.w = higher freq − lowest freq.

B.w ⇒ $(f_0 + f_b) - (f_0 - f_b)$

B.w $= 2f_b$.

$f_b = \frac{1}{T_b}$ ⇒ Maximum freq. is the base band signal.

$f_0 \rightarrow$ carrier freq.

Baud Rate:

The modulated waveform changes at the rate equal to bit rate.

$\boxed{\text{Baud rate} = f_b}$

·Power spectral density:

In BPSK Generation, from the modulator, the complex envelope of a binary PSK wave consists of an inphase component only.

$$g(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} & 0 \leq t \leq T_b \\ 0 & \text{elsewhere} \end{cases}$$

→ The power spectral density of a random binary wave is equal to energy spectral density of the symbol shaping function divided by the symbol duration.

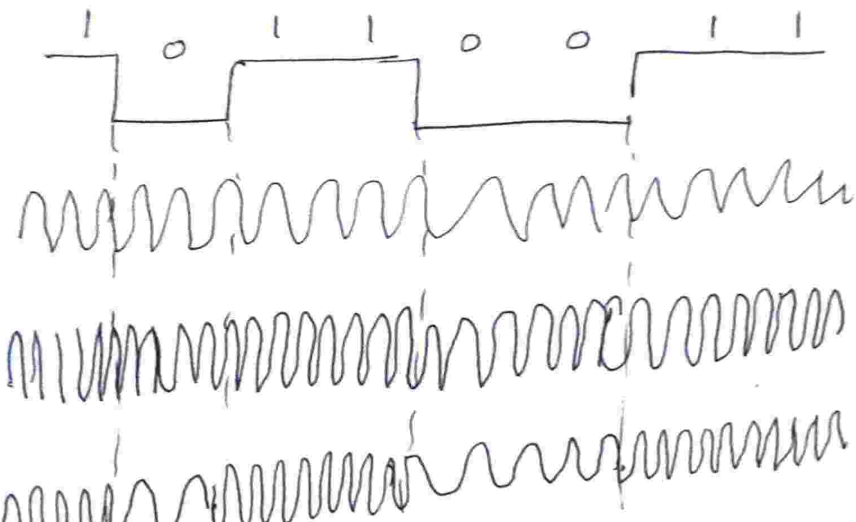
$$S_B(f) = \frac{2E_b \sin^2(\pi T_b f)}{(\pi f T_b)^2}$$

$$\boxed{S_B(f) = 2E_b \operatorname{sinc}^2(T_b f)}$$

Binary Frequency shift keying

Defn

freq. of the carrier sgl is changed in accordance to the message sgl.



→ Low freq carrier

→ High freq carr.

1 → High freq carrier

0 → Low freq carrier

coswht Coswlt coswht coswLt coswht.

$\sqrt{2P_s}\, m(t) \times \cos w_h t$ | $\sqrt{2P_s} \times \cos w_L t$

↓ '1'. ↓ '0' $\Rightarrow \sqrt{P_s T_s} \cdot \sqrt{\frac{P_s}{T_s}} \cos w_L t$

↳ $\phi_2(t)$

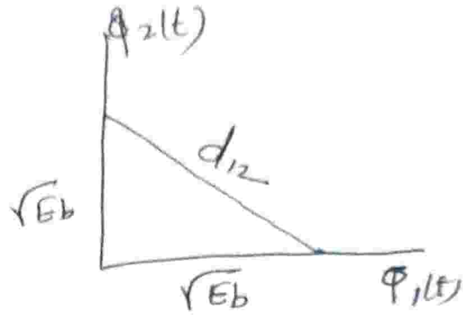
$\sqrt{2P_s} \Rightarrow \sqrt{P_s T_s} \cdot \sqrt{\frac{P_s}{T_s}} \cos w_h t$

↳ Orthogonal function

↳ $\phi_1(t)$

O/P $\Rightarrow s_1(t) + s_2(t)$

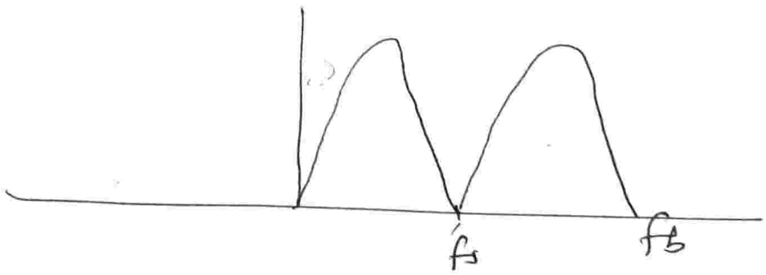
$= \sqrt{P_s T_s}\, \phi_1(t) + \sqrt{P_s T_s}\, \phi_2(t)$



$d_1^2 = \sqrt{E_b^2 + \sqrt{E_b}^2}$

$d_1^2 = \sqrt{2E_b}$ → Eulidian distance.

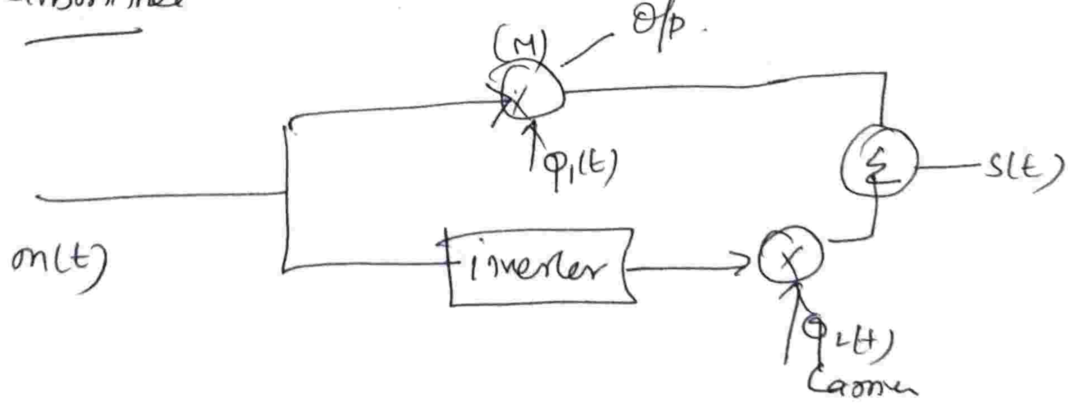
Bandwidth



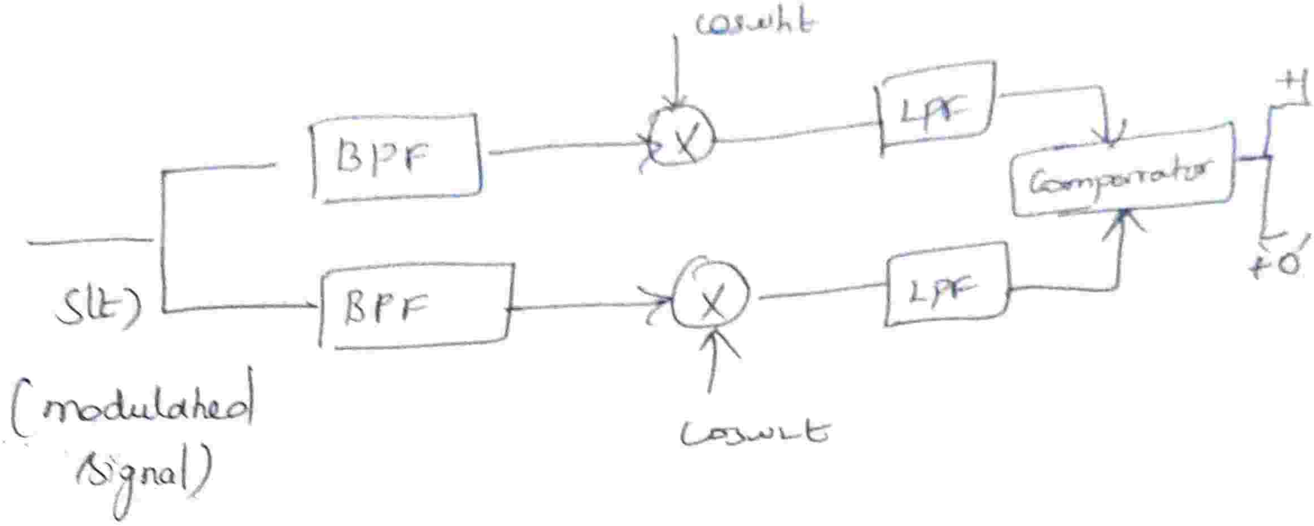
$B.\omega = 2f_b + 2f_b$

$= \underline{4f_b}$

Transmitter



FSK Receiver (coherent)



BPF → Reject the High freq component

Carrier signal is added at the o/p of BPF.

→ o/p of LPF is $\pm A/2$

→ Comparator produce the $\pm A/2$. It is act as a decision device.

~~Quad~~

Quadrature phase shift keying

→ Two bits transmitts → each group has different phases.

Principle:

If two or more bits are combined is some symbols, then the signalling rate is reduced. Therefore the frequency of the carrier required is also reduced. This reduces the channel bandwidth. Thus because of grouping of bits is symbols, the transmission channel B.W is reduced.

In QPSK, two successive bits is the data sequence are grouped together.

In QPSK two successive bits are combined. This combination of two bits forms four distinct symbols. When the symbols is changed to next symbol the phase of the carrier is changed by 45° ($\pi/4$ radians).

Step: 1

→ The I/p binary sequence is first converted to a bipolar NRZ type of signal.

→ This signal is called $b(t)$. It represents binary 1 by $+1V$ and binary '0' by $-1V$.

Step: 2

Demultiplexing into odd and even numbered sequences.

The demultiplexer divides $b(t)$ into two. Separate bit streams of the odd numbered and even numbered bits. $b_e(t)$ represents even numbered sequence and $b_o(t)$ represents odd numbered sequence

→ The symbol duration of both of these odd and even numbered sequence is $2T_b$.

Step: 3

Modulation of quadrature carriers.

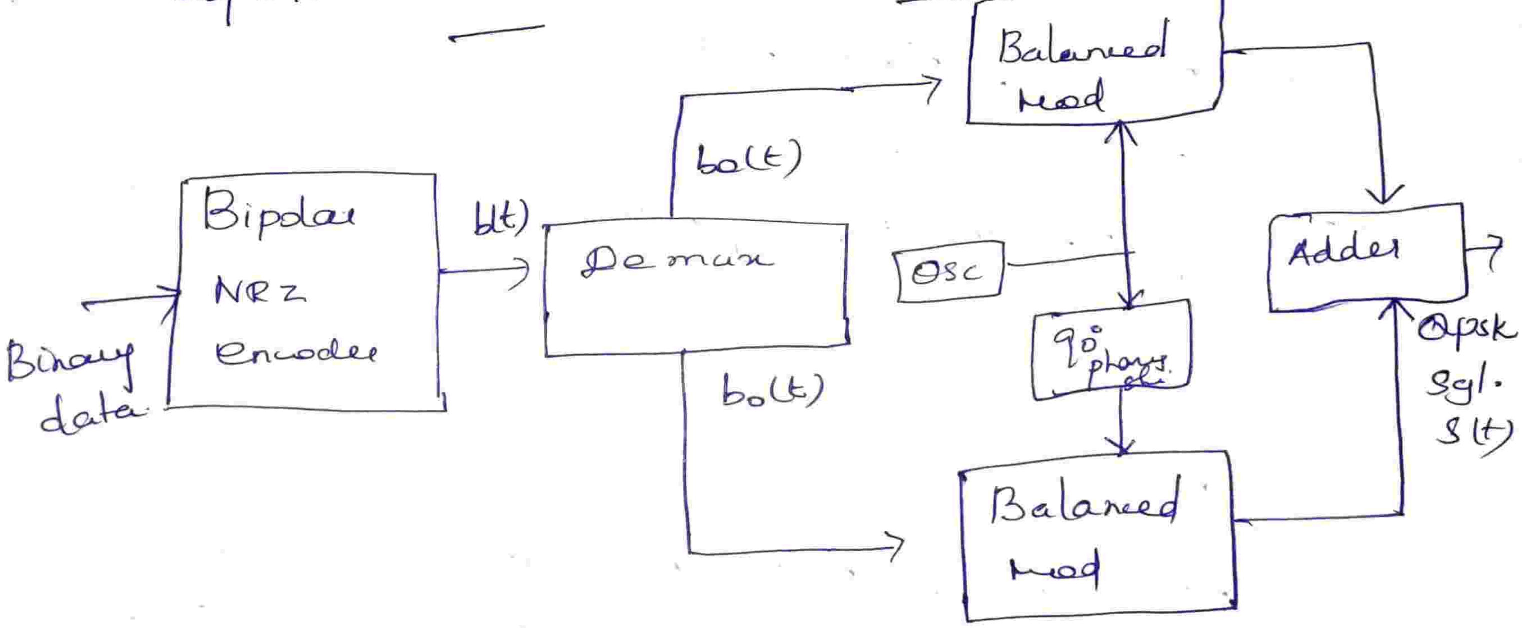
The bit stream $b_e(t)$ modulates carrier $\sqrt{P_s}\cos(2\pi f_0 t)$ and $b_o(t)$ modulates $\sqrt{P_s}\sin(2\pi f_0 t)$.

These modulators are balanced modulator.

The two carriers $\sqrt{P_s}\cos 2\pi f_0 t$ and $\sqrt{P_s}\sin(2\pi f_0 t)$

S.No	Input successive bits		Symbol	phase shift in carrier
i=1	1 (1V)	(-1V) 0	s_1	$\pi/4$
i=2	0 (-1V)	0 (-1V)	s_2	$3\pi/4$
i=3	0 (-1V)	1 (1V)	s_3	$5\pi/4$
i=4	1 (1V)	1 (1V)	s_4	$7\pi/4$.

Qpsk transmitter & Receiver.



Step: 1

Carrier recovery:

The received signal $s(t)$ is first raised to its 4^{th} power ie $s^4(t)$. Then it is passed through a bandpass filter centered around $4f_0$.

→ The o/p of the bandpass filter is a coherent carrier of freq $4f_0$. This is divided by 4. and it gives two coherent quadrature carriers $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 t)$

Step: 2 Synchronous detection:

These coherent carriers are applied to two synchronous demodulator. These synchronous demodulators consist of multiplier and an Integrator

Step: 3 Integration over two bits interval:

The integrator integrates the product signal over two bit interval $(T_s = 2T_b)$.

These carriers are also called quadrature carriers.

$$S_e(t) = b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

$$S_o(t) = b_o(t)\sqrt{P_s}\cos(2\pi f_0 t)$$

$S_e(t)$ and $S_o(t)$ are basically BPSK signals.

$T = 2T_b$

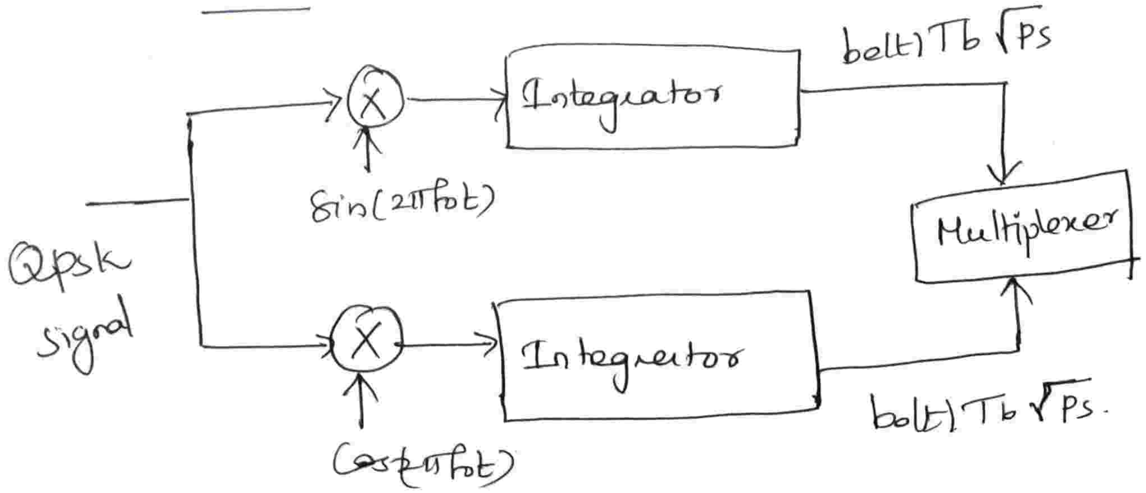
<u>Step: 4</u> Addition of modulated carriers.

The adder add these two signals $b_e(t)$ and $b_o(t)$. The o/p of the adder is Qpsk signal.

$$S(t) = S_o(t) + S_e(t)$$

$$= b_o(t)\sqrt{P_s}\cos(2\pi f_0 t) + b_e(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

Qpsk Receiver



Bandwidth of QPSK

$$B.W = 2 \times \frac{1}{2T_b}$$

$$B.W = f_b.$$

Advantages of QPSK:

→ For the same bit error rate, the bandwidth required by QPSK is reduced to half as compared to BPSK.

→ Because of reduced bandwidth, the information transmission rate of QPSK is higher.

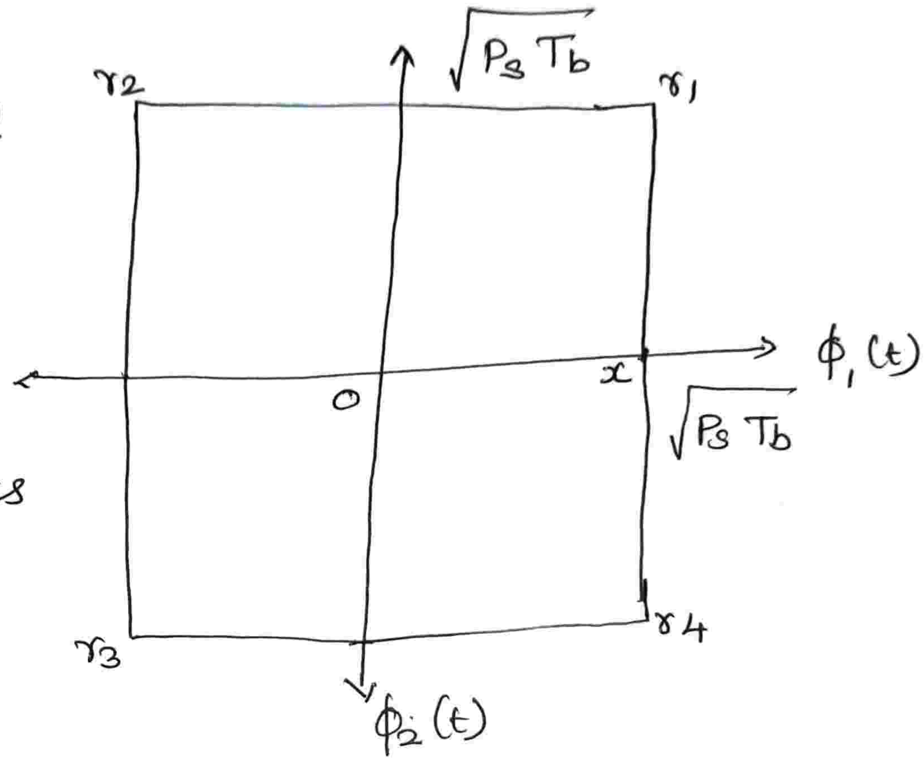
→ Variation in OQPSK amplitude is not much. Hence carrier power almost remains constant.

* Error Probability of QPSK using Signal Space Representation.

For QPSK the transmitted signal is represented as,

$$s(t) = \sqrt{P_s T_b}\ b_o(t)\ \phi_1(t) + \sqrt{P_s T_b}\ b_e(t)\ \phi_2(t)$$

* There are four signal points r_1, r_2, r_3 & r_4.
* The signal point r_1 is obtained due to received vectors $x = \sqrt{P_s T_b}$ & $y = \sqrt{P_s T_b}$.



QPSK Signal Space.

As long as r_1 is the first quadrant decision is correct without any error.

In the presence of noise x & y will include noise n_1 & n_2.

The signal point r_1 remains in first quadrant if

$$0 \le n_1 + x \le \infty \quad \& \quad 0 \le n_2 + y \le \infty \Rightarrow \text{ie } 1^{st} \text{ quadrant.}$$

$$-x \le n_1 \le \infty - x \quad \& \quad -y \le n_2 \le \infty - y$$

$\therefore \; x = y = \sqrt{P_s T_b}$

Since

$$-\sqrt{P_s T_b} \le n_1 \le \infty \quad \& \quad -\sqrt{P_s T_b} \le n_2 \le \infty$$

Thus for Correct decision (S_1 transmitted) n_1 & n_2 will vary from $-\sqrt{P_s T_b}$ to ∞.

Hence

$$P(c/s_1) = P(n_1 > -\sqrt{P_s T_b},\ n > -\sqrt{P_s T_b})$$

Since n_1 & n_2 are uncorrelated

$$P(c/s_1) = P(n_1 > -\sqrt{P_s T_b}) \cdot P(n_2 > -\sqrt{P_s T_b})$$

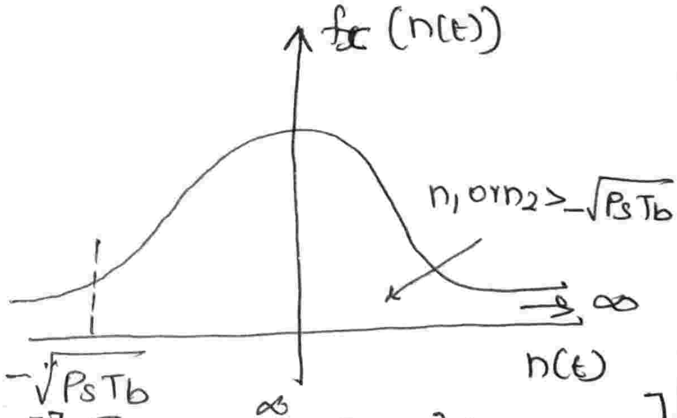
Here n_1 & n_2 are white Gaussian noises with psd of

$\sigma^2 = \frac{N_0}{2}$ ie,

$$f_x(n(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-[n(t)]^2/2\sigma^2}$$

$$= \frac{1}{\sqrt{\pi N_0}} e^{-[n(t)]^2/N_0}, \quad \text{since } \sigma^2 = \frac{N_0}{2}$$

Correct decision probability



$$P(c/s_1) = \left[\frac{1}{\sqrt{\pi N_0}} \int_{-\sqrt{P_s T_b}}^{\infty} e^{-[n(t)]^2/N_0} d[n(t)]\right] \cdot \left[\frac{1}{\sqrt{\pi N_0}} \int_{-\sqrt{P_s T_b}}^{\infty} e^{-[n(t)]^2/N_0} d[n(t)]\right]$$

$$= \left[\frac{1}{\sqrt{\pi N_0}} \int_{-\sqrt{P_s T_b}}^{\infty} e^{-[n(t)]^2/N_0} d[n(t)]\right]^2$$

$$P(c/s_1) = \left[1 - \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{P_s T_b}}^{\infty} e^{-[n(t)]^2/N_0}\, d[n(t)]\right]^2$$

Let $y^2 = \frac{[n(t)]^2}{N_0}$ $y = \frac{n(t)}{\sqrt{N_0}}$, $dy = \frac{d[n(t)]}{\sqrt{N_0}}$

when $n(t) = \sqrt{P_s T_b}$, $y = \frac{\sqrt{P_s T_b}}{\sqrt{N_0}} = \sqrt{\frac{P_s T_b}{N_0}}$

when $n(t) \to \infty$, $y \to \infty$

Hence

$$P(c/s_1) = \left[1 - \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{P_s T_b}{N_0}}}^{\infty} e^{-y^2} \sqrt{N_0}\, dy\right]^2$$

$$= \left[1 - \left(\frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{P_s T_b}{N_0}}}^{\infty} e^{-y^2} dy\right)\right]^2.$$

$$= \left[1 - \frac{1}{2}\left(\frac{2}{\sqrt{\pi}} \int_{\sqrt{\frac{P_s T_b}{N_0}}}^{\infty} e^{-y^2} dy\right)\right]^2$$ By rearranging

Since $\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{\infty} e^{-y^2} dy$, with $u = \sqrt{\frac{P_s T_b}{N_0}}$, above eqn becomes

$$P(c/s_1) = \left[1 - \frac{1}{2}\operatorname{erfc}\sqrt{\frac{P_s T_b}{N_0}}\right]^2 = 1 - \operatorname{erfc}\sqrt{\frac{P_s T_b}{N_0}} + \left(\frac{1}{2}\operatorname{erfc}\sqrt{\frac{P_s T_b}{N_0}}\right)^2$$

Here third term will be very small & it can be neglected

Since s_1, s_2, s_3 & s_4 are equal, above eqn holds for all the symbols.

$$P_c = P(C/s_1) = P(C/s_2) = P(C/s_3) = P(C/s_4)$$

$$= 1 - \text{erfc}\sqrt{\frac{P_s T_b}{N_0}}$$

Error probability

$$P_e = 1 - P_c = 1 - \left(1 - \text{erfc}\sqrt{\frac{P_s T_b}{N_0}}\right)$$

Since $E_b = \sqrt{P_s T_b}$ $\quad \therefore P_e = \text{erfc}\sqrt{\frac{E_b}{N_0}}$

Bit energy E_b & symbol energy E_s are related for QPSK as

$E_b = E_s/2$

$$\boxed{\text{Error probability of QPSK, } P_e = \text{erfc}\sqrt{\frac{E_s}{2N_0}}}$$

we have $\text{erfc}(u) = 2Q(\sqrt{2}u)$

$u = \sqrt{\frac{E_s}{2N_0}}$ above result will be

$$P_e = 2Q\left[\sqrt{2}, \sqrt{\frac{E_s}{2N_0}}\right]$$

$$P_e = 2Q\sqrt{\frac{E_s}{N_0}}$$

Since $E_s = 2E_b$

$$P_e = \text{erfc}\sqrt{\frac{E_b}{N_0}} = 2Q\sqrt{\frac{2E_b}{N_0}}$$

Above eqn gives symbol error probability.

Quadrature Amplitude Shift Keying (QASK) or Quadrature Amplitude Modulation (QAM)

It can be defined as a consolidation of ASK and PSK, so that a max. difference between each signal unit (bit, dibit, tribit and so on) is accomplished.

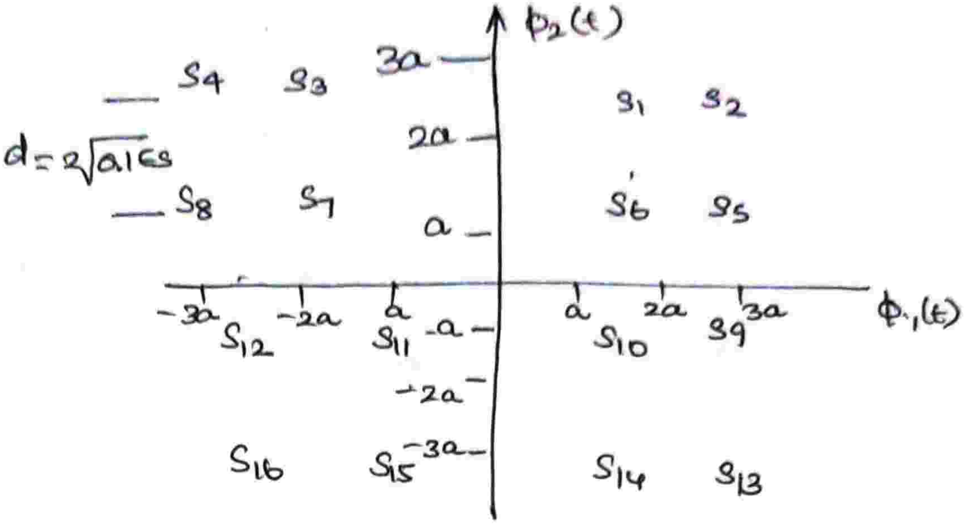
As with analog modulation, there are three parameters of the carrier wave to vary and therefore three basic types of shift keyings.

Amplitude shift keying (ASK)
Frequency Shift keying (FSK)
Phase shift keying (PSK)

Euclidean distance in QPSK.

Normalized Euclidean distance d 2min for QPSK modulation with $\gamma 2 = \pi/4$

Let us consider the case of 4 bit symbol. Then there will be $2^4 = 16$ possible symbols. In the QASK System such 16 symbols are represented geometrically as shown in fig.



The distance from the neighbouring points is $d = 2a$.

Let the signals be equally. Then the avg. associated with the signal can be obtained

$$E_s = \frac{1}{4}\left[(a^2+a^2)+(9a^2+a^2)+(a^2+9a^2)+(9a^2+9a^2)\right] = 10a^2$$

$$a = \sqrt{0.1E_s}$$

Since $d = 2a$ we have

$$d = 2\sqrt{0.1E_s} = \sqrt{0.4E_s}.$$

This gives the distance between two signal points in 16 QASK. In each symbol there are 4 bits.

$$E_s = 4E_b$$

$$\therefore\ d = \sqrt{0.4 \times 4E_b} = \sqrt{1.6E_b}$$

The distance for QPSK

$$d_{QPSK} = 2\sqrt{E_b} = \sqrt{4E_b}$$

& the distance for 16 ary PSK

$$d_{16\,PSK} = 2\sqrt{E_s}\ \sin \pi/16$$

$$= 2\sqrt{4E_b}\ \sin \pi/16$$

$$= 2\sqrt{0.15E_b} = \sqrt{0.6E_b}$$

Thus the distance of 16 QASK is greater than 16-ary PSK

Transmitter & Receiver of QASK

4-bit symbol

The signal represent

$$S(t) = K_1 a\, \phi_1(t) + K_2 a\, \phi_2(t)$$

Here K_1 & K_2 will take values of ± 1 or ± 3 $\phi_1(t)$ & $\phi_2(t)$

are orthogonal carriers

$$\phi_1(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_0 t)$$

&

$$\phi_2(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_0 t)$$

from Es eqn W.k.T

$$a = \sqrt{0.1 E_s}$$

∴ we can write

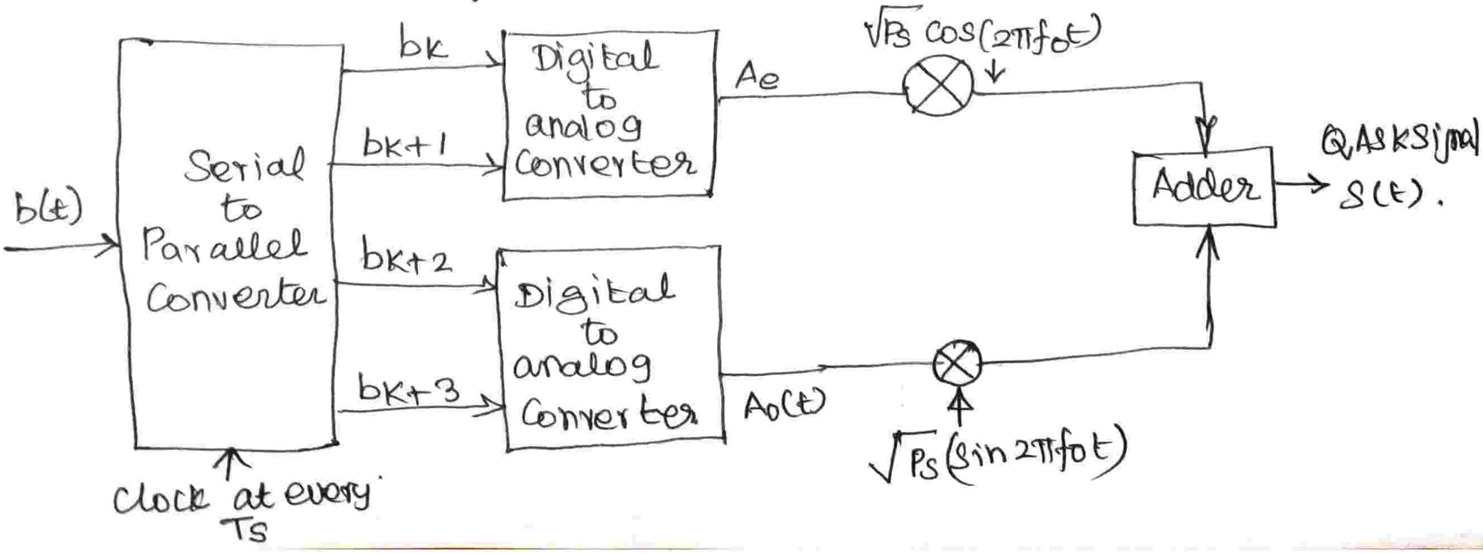
$$S(t) = K_1 \sqrt{0.2 \frac{E_s}{T_s}} \cos(2\pi f_0 t) + K_2 \sqrt{0.2 \frac{E_s}{T_s}} \sin(2\pi f_0 t)$$

W.K.T $E_s = P_s T_s$

$$\frac{E_s}{T_s} = P_s$$

then the above eqn becomes

$$S(t) = K_1 \sqrt{0.2 P_s} \cos(2\pi f_0 t) + K_2 \sqrt{0.2 P_s} \sin(2\pi f_0 t)$$



Generation of QASK signal

$$S(t) = A_e(t)\sqrt{P_s}\cos(2\pi f_0 t) + A_o(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

Receiver of QASK signal.

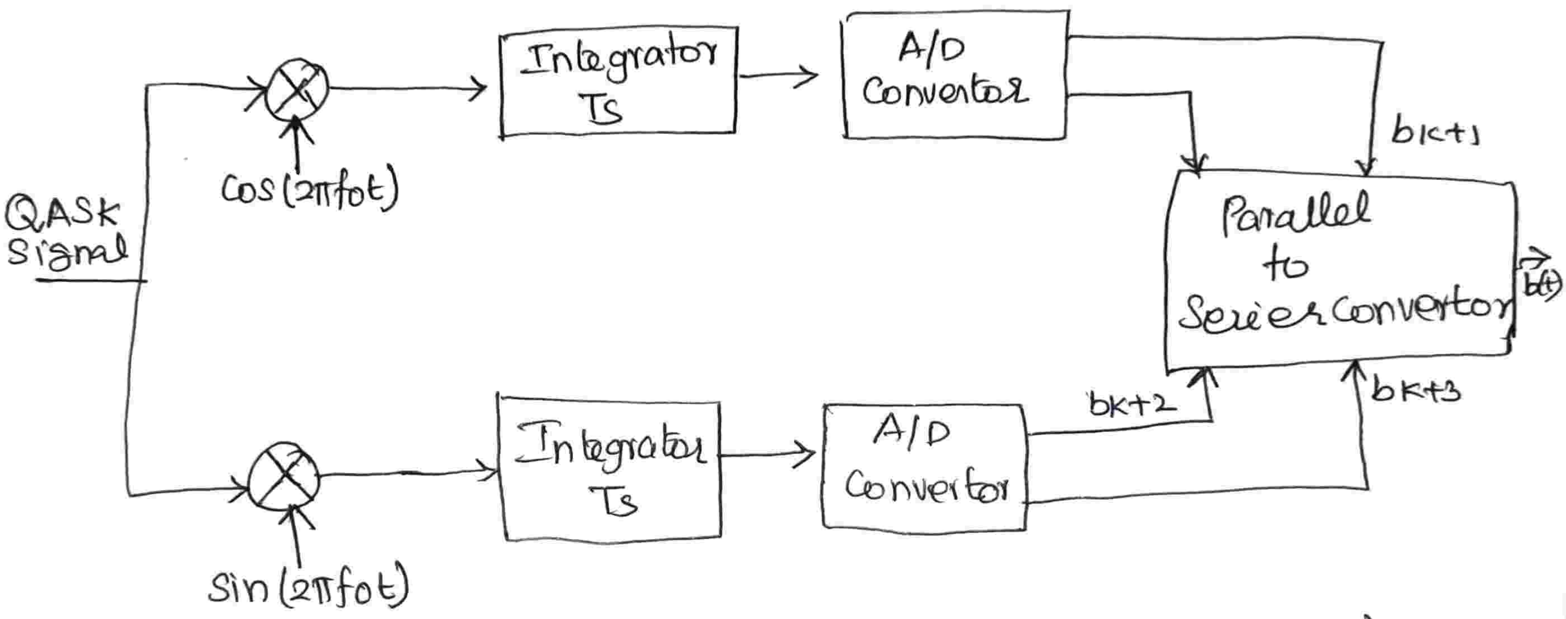


Fig shows the receiver of 16-QASK (4 bit QASK)
The Carrier recovery circuit obtain quadrature Carriers from received QASK Signal. there Carriers are $\cos(2\pi f_0 t)$ & $\sin(2\pi f_0 t)$.

Quadrature Amplitude Shift keying.

(QASK) (or) Quadrature Amplitude Modulation

If the amplitude of the signal is also varied, then the points will lie inside the circle also. on the signal space diagram.

This further increases the noise immunity of the s/m. Such s/m involves phase as well as Amplitude shift keying.

It is called quadrature Amplitude phase shift keying (QASK).

Transmitter and Receiver of QASK:

$$s(t) = k_1 a \phi_1(t) + k_2 a \phi_2(t)$$

k_1 & k_2 $\Longrightarrow$ values of ± 1

$$\phi_1(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_0 t$$

$$\phi_2(t) = \sqrt{2/T_b} \sin 2\pi f_0 t$$

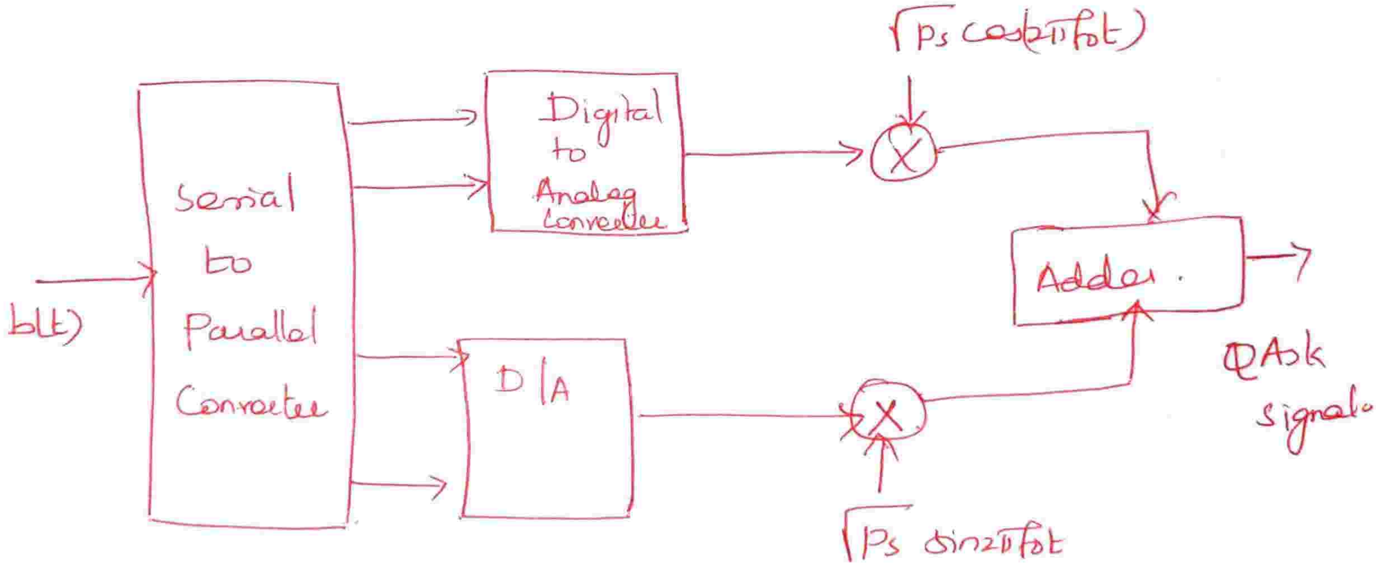
$$a = \sqrt{0.1 E_s}$$

$$s(t) = k_1 \sqrt{0.2 \frac{E_s}{T_s}} \cos 2\pi f_0 t + k_2 \sqrt{0.2 \frac{E_s}{T_s}} \sin(2\pi f_0 t)$$

$$E_s = P_s T_s.$$

$$\frac{E_s}{T_s} = P_s.$$

$$s(t) = k_1 \sqrt{0.2 P_s} \cos(2\pi f_0 t) + k_2 \sqrt{0.2 P_s} \sin(2\pi f_0 t)$$



→ The i/p bit stream is applied to a serial to parallel converter.

→ Four successive bits are applied to the digital to analog converter.

These bits are applied after every T_s second. T_s is the symbol period and $T_s = 4T_b$

Bits b_k and b_{k+1} are applied to upper digital to analog converter and b_{k+2} and b_{k+3} are applied to lower digital to analog converter.

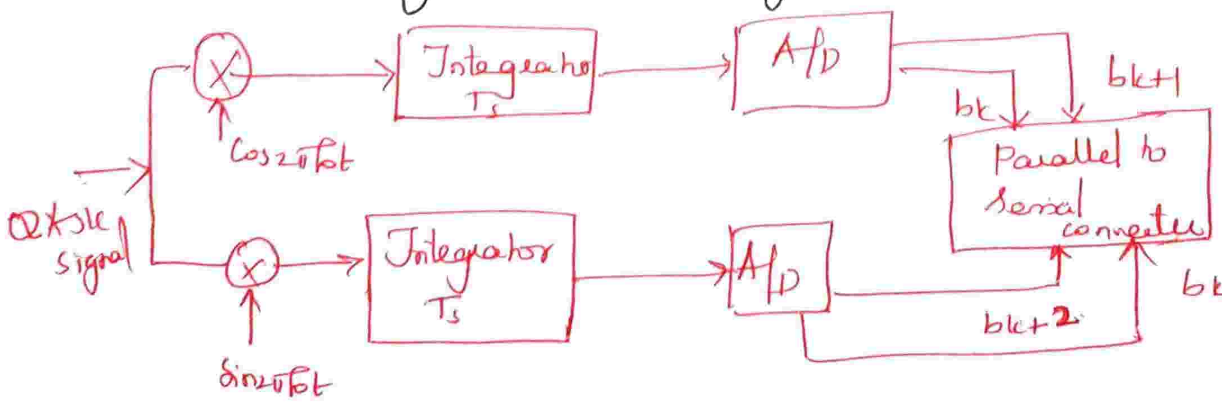
Depending upon two i/p bits, the o/p of analog digital to analog converter takes four levels.

→ $A_e(t)$ and $A_o(t)$. i/p, carrier signal $\sqrt{P_s}\cos 2\pi f_0 t$ $\sqrt{P_s}\sin 2\pi f_0 t$.

$$S(t) = A_e(t)\sqrt{P_s}\cos 2\pi f_0 t + A_o(t)\sqrt{P_s}\sin(2\pi f_0 t)$$

$A_e(t) = \pm\sqrt{0.2}$

Receiver of QASK signal:



→ The carrier recovery ckt obtains quadrature carrier from received QASK signal. These carrier are $\cos 2\pi f_0 t$ and $\sin(2\pi f_0 t)$

→ The Integrator integrate the multiplied signals over one symbol period. The o/p of integrator at sampling period give $A_e(t)$ and $A_o(t)$.

→ The analog to digital converter give the four bits b_k, b_{k+1}, b_{k+2}, b_{k+3}.

→ The parallel to serial converter then generates the bit sequence $b(t)$.

→ The inphase and quadrature coherent carriers are multiplied with QASK signal.

Bandwidth of QASK signal:

$$B.\omega = f_s - (-f_s) = 2f_s$$

$$= 2/T_s$$

$$= \frac{2}{NT_b}$$

$$\boxed{B.\omega = \frac{2f_b}{N}}$$

Power spectral density.

$$S(f) = P_s T_s \left[\frac{\sin(\pi f T_s)}{\pi f T_s}\right]^2$$

Differential phase shift keying (DPSK)

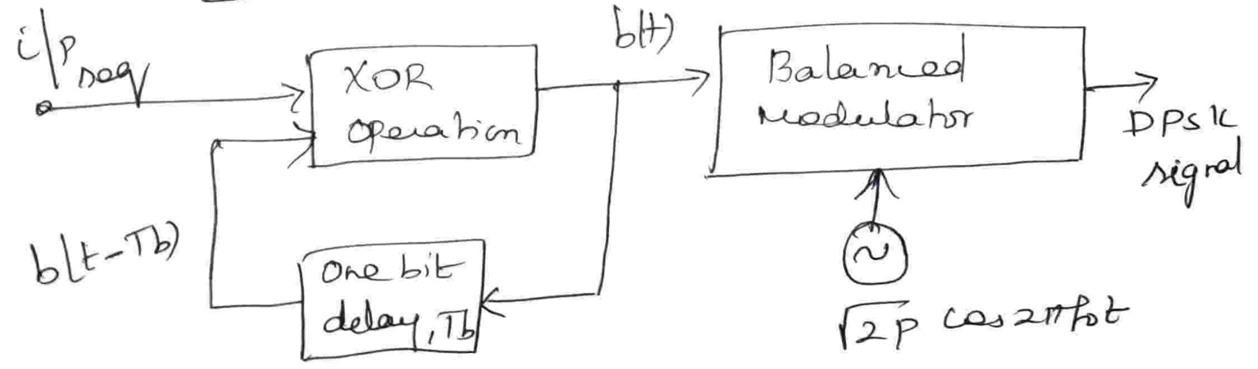
Principle:

DPSK is differentially coherent modulation method. DPSK does not need a synchronous carrier at the demodulator.

→ The i/p sequence of binary bits is modified such that the next bit depends upon the previous bit. Therefore in the receiver the previous received bits are used to detect the present bit.

DPSK transmitter and Receiver.

DPSK Transmitter



→ $b(t) = d(t) \oplus b(t-T_b)$. The initial value of $b(t-T_b)$ is assumed zero.

→ The differentially encoded signal $b(t)$ then performs BPSK modulation of the carrier $\sqrt{2P}\cos(2\pi f_0 t)$

→ Always two successive bits of $d(t)$ are checked for any change of level. Hence the one symbol has two bits

Symbol duration (T) = duration of 2 bits

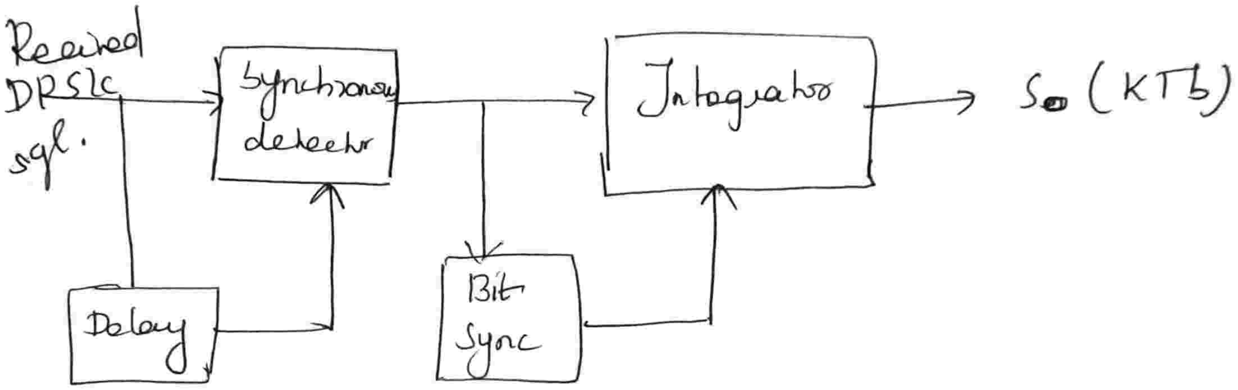
$$T = 2T_b.$$

→ The sequence $b(t)$ is applied to a balanced Modulator The balanced modulator is also supplied with a carrier $\sqrt{2P}\cos(2\pi f_0 t)$

$$s(t) = b(t)\sqrt{2P}\cos 2\pi f_0 t$$

$$= \pm\sqrt{2P}\cos 2\pi f_0 t$$

DPSK Receiver



Operation of Receiver:

1. phase shift in received signal: During the transmission, the DPSK signal undergoes some phase shift θ. Therefore the signal received at the i/p of the receiver is

$$\text{Rxd signal} = b(t)\sqrt{2P}\cos(2\pi f_0 t + \theta)$$

2. Synchronous detector (or) Multiplier o/p:
This signal is multiplied with its delayed version of one bit.

$$\text{Multiplier o/p} = b(t)\, b(t-T_b)\, P\left[\cos 2\pi n + \cos 4\pi f_0\left[t - \frac{T_b}{2}\right] + 2\theta\right]$$

$\cos 2\pi n = 1$

Multiplier o/p $\Rightarrow$ $b(t)\, b(t-T_b) P + b(t)\, b(t-T_b)\, P$ $\cancel{\cos}$

$$\cos\left[4\pi f_0\left(t - T_b/2\right]\right) + 2\theta$$

③ Integrator

The Integration of the 2nd term will be zero. Since it is integration of carrier over one bit duration.

$$s_0(kT_b) = b(kT_b)\, b\left[(k-1)T_b\right] \cdot P\left[kT_b - (k-1)T_b\right]$$

Multiplier o/p

$$= b(t)\, b(t-T_b)\, (2P) \cos(2\pi f_0 t + \theta)$$
$$\cos[2\pi f_0 [t - T_b) + \theta]$$

$A = 2\pi f_0 t + \theta \qquad B = 2\pi f_0 (t - T_b) + \theta$

Multiplier o/p $\Rightarrow b(t)\ b(t-T_b)\ P \left\{ \cos 2\pi f_0 T_b + \right.$
$$\cos\left[4\pi f_0 \left(t - \frac{t_b}{2}\right)\right] + 2\theta$$

$f_0 \rightarrow$ carrier frequency

$T_b \Rightarrow$ One bit period.

$T_b \Rightarrow$ Contains integral number of cycles of 'f_0'

$$f_b = \frac{1}{T_b}.$$

T_b Contains 'n' cycles.

$$f_0 = n f_b = f_0 = n / T_b$$

$$\boxed{f_0 T_b = n}$$

$$b(t)\, b(t - T_b) = -1V$$

$$S_o(kT_b) \begin{cases} -PT_b & d(t) = 1 \\ +PT_b & d(t) = 0. \end{cases}$$

B.ω of DPSK

$$T = 2T_b$$

$$B.\omega = 2/T = 1/T_b$$

$$\boxed{B.\omega \quad f_b}$$

Carrier synchronization using Mth Power loop

Block diagram

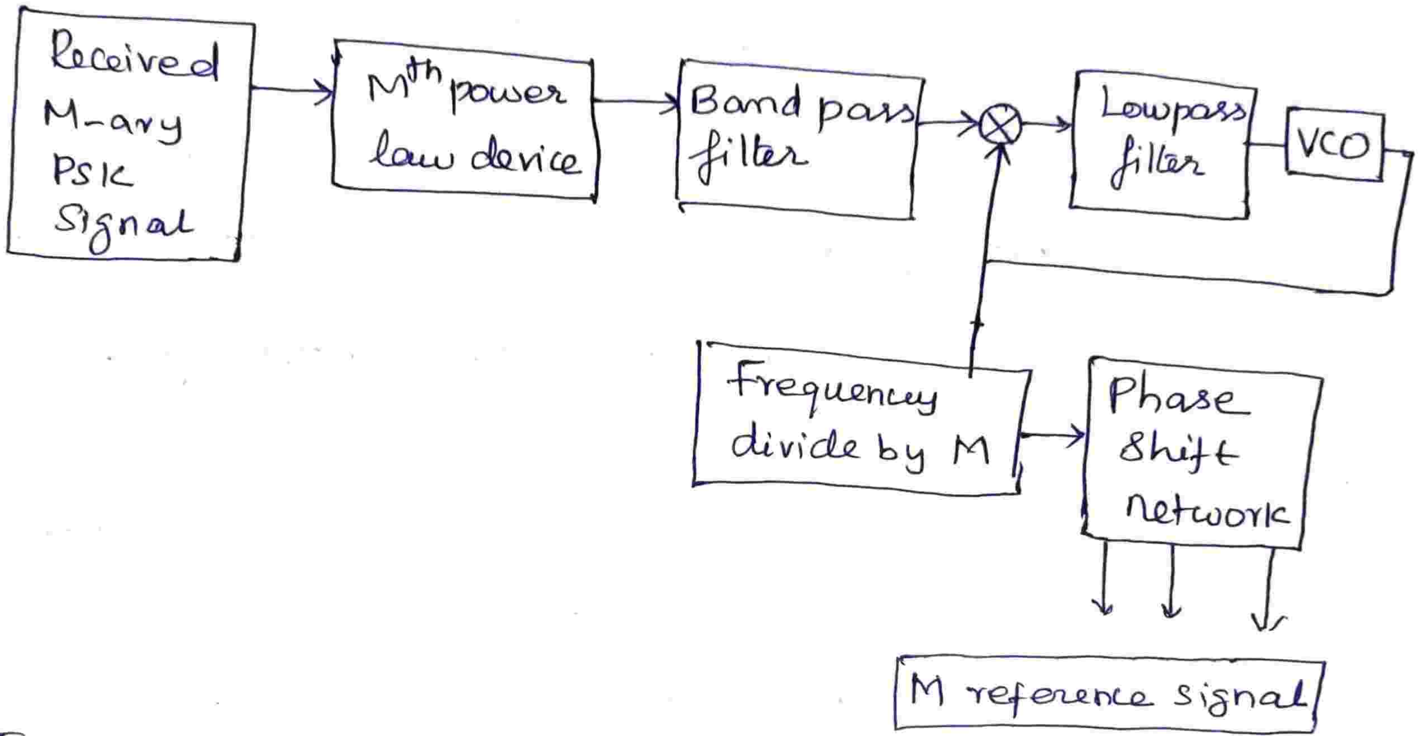


Figure shows, the block diagram of Carrier recovery circuit for M-ary PSK*. This circuit is called the Mth power loop When M=2, then it is called Squaring loop. When M=2, the M-ary PSK is then Called as binary PSK. The input signal is first raised to the Mth power by the mth power law device with the frequency fc.

The phase locked loop Consists of a phase detector, low-pass filter and VCO. The phase locked loop tracks the carrier frequency. Then the output of a Voltage Controlled oscillator (VCO) is the Carrier frequency. The output frequency of VCO is divided by M

This is done because the Mth Power of the input Signal multiples carrier frequency by M. The phase Shift network then separates 'M' reference signals for the 'M' correlation receivers. In the technique the power of the input signal is raised to some power say 'M'.

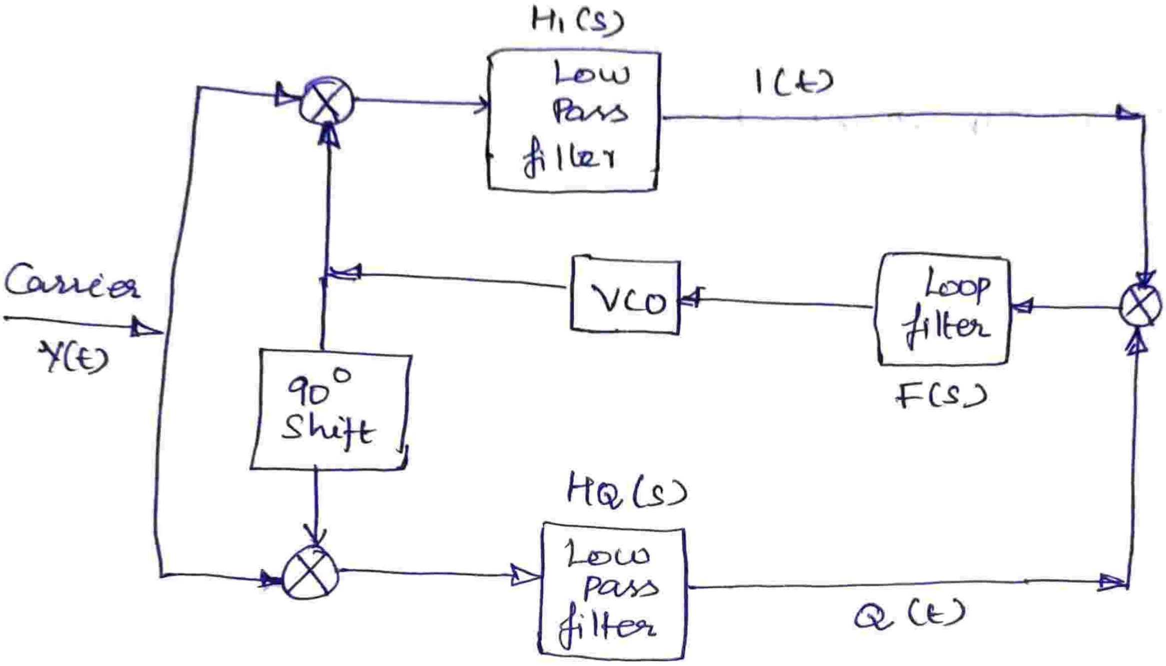
Let us say M=2; then the input signal is squared. Because of this, the sign of the recovered carrier is always independent of sign of the input signal carrier since it is required. Therefore there can be 180 degree error in the output.

Coastas loop.

This loop design is important because it eliminates the square law device, which can be difficult to implement at carrier frequency and replaces it with a multiplier and relatively simple low pass filters.

The problem in this loop is to important the two LPF identical. This problem can be solved by using the digital filter.

The decision as to whether to implement a Costas loop or the classical design of amount to a decision between the difficulty of implementing the Squaring device and the difficulty of implementing closely matched arm filters.



$\frac{1}{2}\sqrt{\frac{2E}{T}}\cos(\varphi-\theta)$ & $\frac{1}{2}\sqrt{\frac{2E}{T}}\sin(\varphi-\theta)$. The multiplier output is given as,

$$V_m = \frac{1}{4}\times 2E/T \sin(\varphi-\theta)\cos(\varphi-\theta)$$

$$= E/2T \cdot \frac{1}{2}\sin^2(\varphi-\theta)$$

$$V_m = E/4T \sin^2(\varphi-\theta)$$

The power 'P' of the signal over the period T is given by $P = E/T$

Therefore eqn V_m can be written as

$$V_m = P/4 \sin^2(\varphi-\theta)$$

If there is some difference between the VCO frequency and the input carrier frequency then the phase difference $(\varphi-\theta)$ is changed proportionally. The change in $(\varphi-\theta)$ causes V_m to increase or decrease VCO frequency such that synchronization is maintained.

UNIT: V

ERROR CONTROL CODING

Introduction

Information is transmitted through the Channel with a rate 'R' called information rate.

Shannon's theorem says that it is possible to transmit information with an arbitrarily small probability of error provided that information rate 'R' is less than or equal to a rate 'C' called channel capacity.

Channel capacity is the maximum information rate with which the error probability is within the tolerable limits.

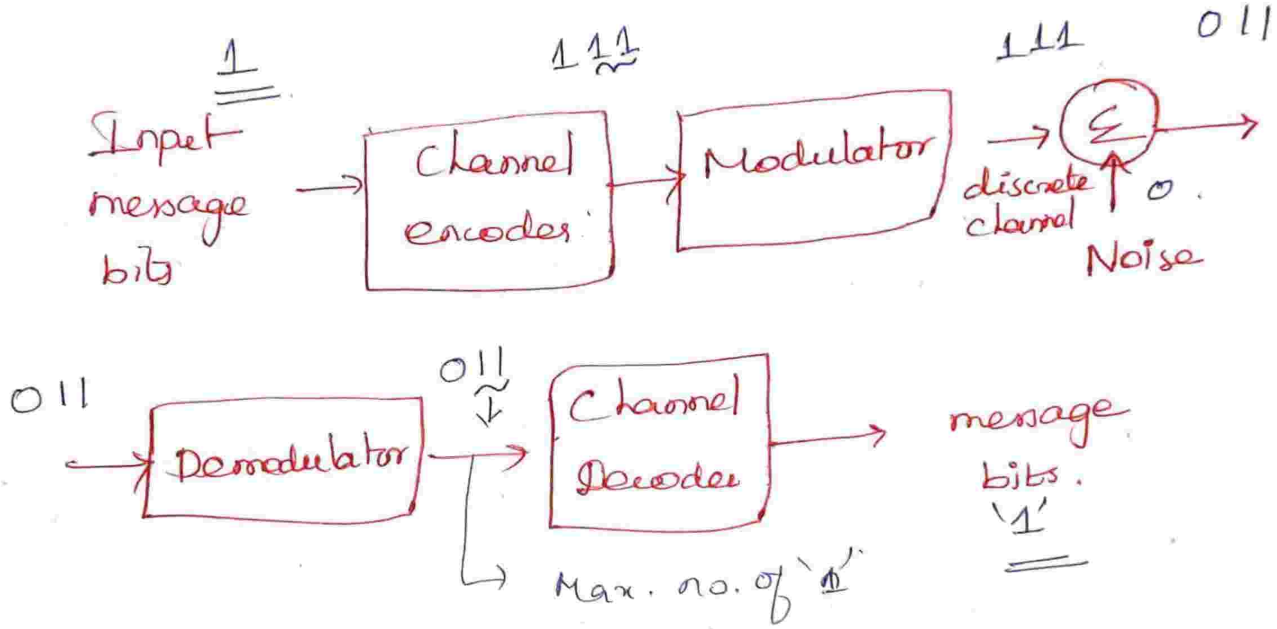
Statement:

Given a source of 'M' equally likely messages, with M>>1, which is generating information at a rate 'R'.

$$\boxed{R \leq C}$$

Negative statement of channel coding theorem.

$R > C$



Error Control Codes:

Block Codes:

These codes consists of 'n' number of bits in one block (or) codeword. This codeword consists of 'k' message bits (n−k) redundant bits. Such block codes are called (n, k) block codes.

Block Length:

The no. of bits 'n' after coding is called the block length of the code.

Code rate:

The ratio of message bits (k) and the encoder o/p bits (n) is called code rate.

$r = k/n$

$0 < r < 1$

Channel data rate:

If the bit rate at the i/p of encoder is R_s, then the channel data rate will be

Channel data rate $(R_0) = \frac{n}{k} R_s$.

Systematic code:

It is able to separate the message bit and check bit in the codeword. that codeword is called systematic code.

Non systematic code:

It is not possible to identify the message bits and checkbits.

Linear code:

A code is linear if the sum of the two code vectors produces another code vector.

$$X = \{m_1, m_2, m_3 \ldots m_k, c_1, c_2, \ldots c_q\}$$

$q = n - k.$

$q \rightarrow$ no. of redundant bit.

$$X = (M | c)$$

Ex: $\{1, 00101\}$

$n = 6$

$k \Rightarrow$ Msg. bits.

Convolutional codes:

The coding operation is discrete time convolution of i/p sequence with the impulse response of the encoder.

→ The convolutional encoder accepts the message bits continuously and generates the encoded seq. continuously.

Linear codes:

If the two codewords of the linear codes are added by modulo - 2 arithmetic, then it produces third codeword in the code.

Nonlinear codes:

Addition of the nonlinear codewords does not necessarily produce third codeword.

Types of error control.

(i) Forward acting error correction

→ The errors are detected and corrected by proper coding technique at the receiver

→ The check bits or redundant bits are used by the receiver to detect and correct errors.

→ error detection and correction capability of the receiver depends upon number of redundant bits in the transmitted message.

→ The forward Acting error correction is faster, but overall probability of error is higher. because some of the error cannot be corrected.

(ii) Error detection with retransmission:

→ decoder checks the i/p sequence, when it detect any error, it discards that part of the sequence and requests the transmitter for retransmission.

→ The transmitter then again transmits the part of the sequence in which error was detected.

→ The decoder does not correct the errors. It just detect the errors and send requests to transmitter.

Types of errors:

1. Random error:

These errors are created due to white gaussian noise in the channel.

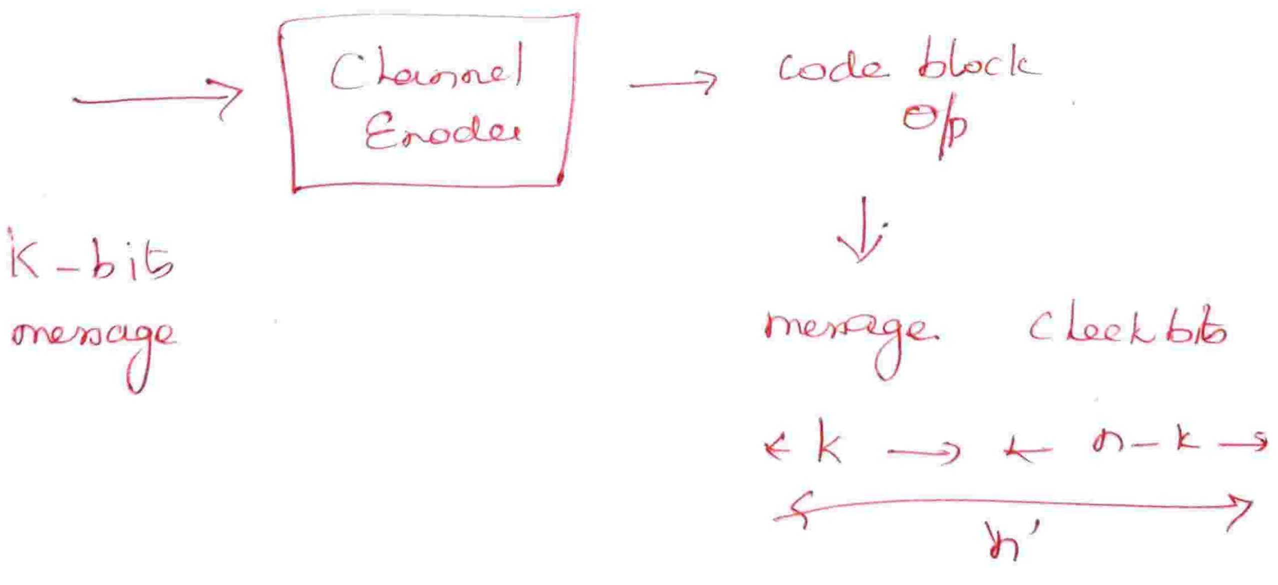
→ The errors generated due to white gaussian noise in the particular interval. does not affect the performance of the s/m. in the subsequent intervals.

(ii) Burst errors:

→ These errors are generated due to compulsive noise in the channel. These impulse noise are generated due to lightning and switching transients. These noise bursts affect several successive symbols.

Codeword:

The encoded block of 'n' bits is called a codeword. It contains message bits and redundant bits.



Matrix Description of Linear Block codes.

$$X = MG$$

$X \rightarrow$ code vector of $1 \times n$ size

$M \rightarrow$ Message vector of $1 \times k$.

$G \rightarrow$ Generator Matrix $k \times n$. sizes

$$G = \left[I_k \mid P_{k \times q} \right]_{k \times n}$$

$I_k \rightarrow$ Identity Matrix

$P_{k \times q} \rightarrow$ Sub matrix.

code vector $\rightarrow$ $C = mP$.

$$(C_1, C_2, \ldots C_q)_{1 \times q} = \left[m_1 \; m_2 \; \ldots \; m_k \right]_{1 \times k} \begin{bmatrix} P_{11} & P_{12} & \ldots & P_{1q} \\ P_{21} & & & \\ \vdots & & & \\ P_{k1} & P_{k2} & & P_{kq} \end{bmatrix}$$

$$C_1 = m_1 P_{11} \oplus m_2 P_{21} \oplus m_3 P_{31} \oplus \ldots m_k.$$

$$C_2 = m_2 P_{11} \oplus$$

Parity check Matrix (H)

$H = P^T / I$

Hamming codes:

Hamming Codes are (n,k) linear block codes

→ Number of check bits $q \geq 3$.

→ Block length $n = 2^q - 1$

→ Number of message bits $K = n - q$

→ Minimum distance $d_{min} \simeq 3$.

Syndrome decoding:

→ This method to correct errors in linear block coding pbm: 1

① The Generator Matrix for a (6, 3) block code ~~to~~ is given below. Find all code vectors of this code.

$$G = \begin{bmatrix} 1 & 0 & 0 & : & 0 & 1 & 1 \\ 0 & 1 & 0 & : & 1 & 0 & 1 \\ 0 & 0 & 1 & : & 1 & 1 & 0 \end{bmatrix}$$

(i) determine P Sub matrix from generator Matrix

(ii) Obtain equation for check bits using $C = MP$

(iii) determine Check bits for every message vector

(iv) What is the minimum distance b/w the code vectors?

(v) How many errors and can be detected? How many errors are can be corrected?

(vi) Decode 001010.

$(n, k) \rightarrow (6, 3)$

$q = n - k$

(i) $G = I_{k \times k} \mid P_{k \times q}$ $\boxed{q = 3.}$

$$[P]_{k \times q} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(ii) Check bits $C = MP$.

$M \rightarrow$ message bits.

$P \rightarrow$ Sub matrix.

$q = 3 = m = 3$ bit]

$q = 4 = m = 4$ bit

$$[c_1, c_2 \ldots c_q]_{1 \times q} = [m_1, m_2, \ldots m_q] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(iii) d_{min} = minimum non zero code length weight.

Soln

$$I_k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{k \times q} = P_{3 \times 3} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

(ii) check bits equation:

S. No	Bits of message vector in one block		
	m_1	m_2	m_3
1.	0	0	0
2.	0	0	1
3.	0	1	0
4.	0	1	1
5.	1	0	0
6.	1	0	1
7.	1	1	0
8.	1	1	1

$$[C_1 \quad C_2 \quad C_3] = [m_1 \quad m_2 \quad m_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$C_1 = (0 \times m_1) \oplus (1 \times m_2) \oplus (1 \times m_3)$$

$$C_2 = (m_1 \times 1) \oplus (0 \times m_2) \oplus (1 \times m_3)$$

$$C_3 = (m_1 \times 1) \oplus (1 \times m_2) \oplus (0 \times m_3)$$

$$C_1 = m_2 \oplus m_3$$

$$C_2 = m_1 \oplus m_3$$

$$C_3 = m_1 \oplus m_2$$

(iii) To determine check bits and codevectors for every message vector.

Consider first block of $(m_1, m_2, m_3) = 0, 0, 0$

weight of code vector no. of 1's.

S.no	Message bit m_1	m_2	m_3	Check bits C_1	C_2	C_3	complete codevector m_1	m_2	m_3	C_1	C_2	C_3
1.	0	0	0	0	0	0						
2.	0	0	1	1	1	0						
3	0	1	0	1	0	1						
4	0	1	1	0	1	1						
5	1	0	0	0	1	1						

$d_{min} = 3.$

(V) $d_{min} \geq s+1$ detect

$d_{min} \geq 2t+1$ correct

Detection

$d_{min} \geq s+1$

$3 \geq s+1$

$3-1 \geq s.$

$\boxed{2 \geq s.}$

Correction

$d_{min} \geq 2t+1$

$3 \geq 2t+1$

$2 \geq 2t$

$2/2 \geq t$

$1 \geq t$

one error corrected.

Decode: 001010 → 'y'.

$Syn = y.H^T$ — (a).

$S = E.H^T$

X → transmit

Y → Receive

Error vector = X − Y.

Given

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$$G = [I_k : P_{k \times q}]$$

$$H = [P^T : I_k]$$

$$P^T \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H^T \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Syndrome = $y.H^T$

Error vector = n value $\boxed{n = 6}$

Syndrome decoding table

S.No	Error vector	Syndrome
1	0 0 0 0 0 0	0 0 0
2	1 0 0 0 0 0	0 1 1
3	0 1 0 0 0 0	1 0 1
4	0 0 1 0 0 0	1 1 0
5	0 0 0 1 0 0	1 0 0
6	0 0 0 0 1 0	0 1 0
7	0 0 0 0 0 1	0 0 1

$S = EH^T$

$$[1\,0\,0\,0\,0\,0] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$S = \underline{\underline{011}}$

~~YoH~~^{T} $S = yH^T$

$$[0\,0\,1\,0\,1\,0] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow [1\,0\,0]$

000100
001010
001110

Convolutional Codes:

① Cyclic codes and Linear block codes. block codes Data encoded in blocks.

② Convolutional codes → Sequence of i/p bit by bit transmission (Encoding)

③ Time domain and Matrix Approach }, Viterbi Algorithm

→ Convolutional Coding is done by combining the fixed number of i/p bits. The i/p bits are Stored in the fixed length shift register and they are combined with the help of mod-2 adders. This operation is equivalent to binary convolution and hence it is called Convolutional coding.

→ Convolutional encoder operates on the incoming message sequence continuously in a Serial manner.

→ The encoder of a binary Convolutional code with rate 1/n, measured in bits per symbol.

An L-bit message sequence produces coded o/p sequence of length n(L+M) bits

The code rate

$$r = \frac{L}{n(L+M)} \text{ bits/symbol}$$

L >> M, Hence the code rate

$$r \simeq \frac{1}{n} \text{ bits/symbol}$$

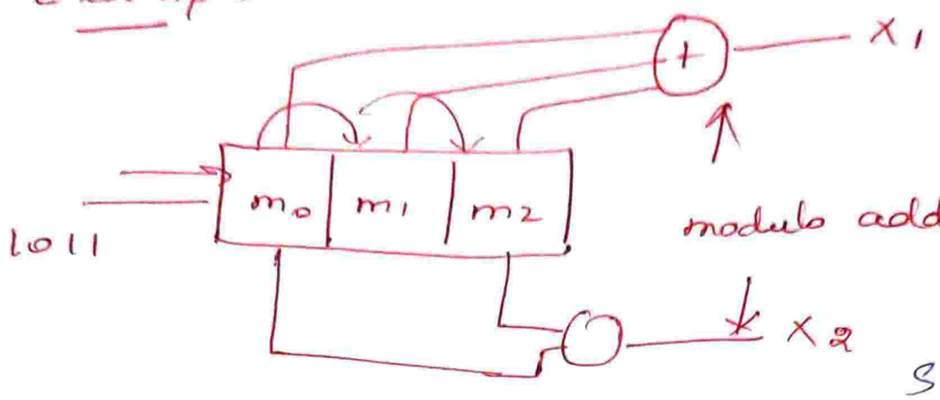
→ present and previous bits are used to encode the sequence.

Constant length:

No. of shifts over which single msg bit influence the encoder o/p.

bits

Example



Shifting operation

000 → 100 → 110 → 011 → 101 → 010 → 001 →

$X_1 = m_0 \oplus m_1 \oplus m_2$

$X_2 = m_0 \oplus m_2$

Encoded sequence : $X_1\ X_2 \mid X_1\ X_2 \mid X_1\ X_2\ X_1\ X_2 \ldots$

Shift Shift

m_0	m_1	m_2	X_1	X_2
0	0	0	0	0
1	0	0	1	1
1	1	0	0	1
0	1	1	0	1
1	0	1	0	0
0	1	0	1	0
0	0	1	1	1

Encoded seq = 00110101 00 101ϕ

No. of states = 2^{N-1}

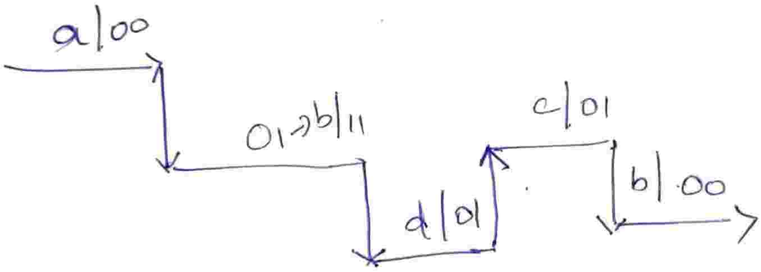
$2^{3-1} = 2^2 = 4$ state

m_2	m_1	
0	0	$\rightarrow$ a
0	1	$\rightarrow$ b
1	0	$\rightarrow$ c
1	1	$\rightarrow$ d.

Code Tree

Rules $\rightarrow$ 1 (i/p) $\rightarrow$ move down

$\rightarrow$ 0 (i/p) $\rightarrow$ move up.



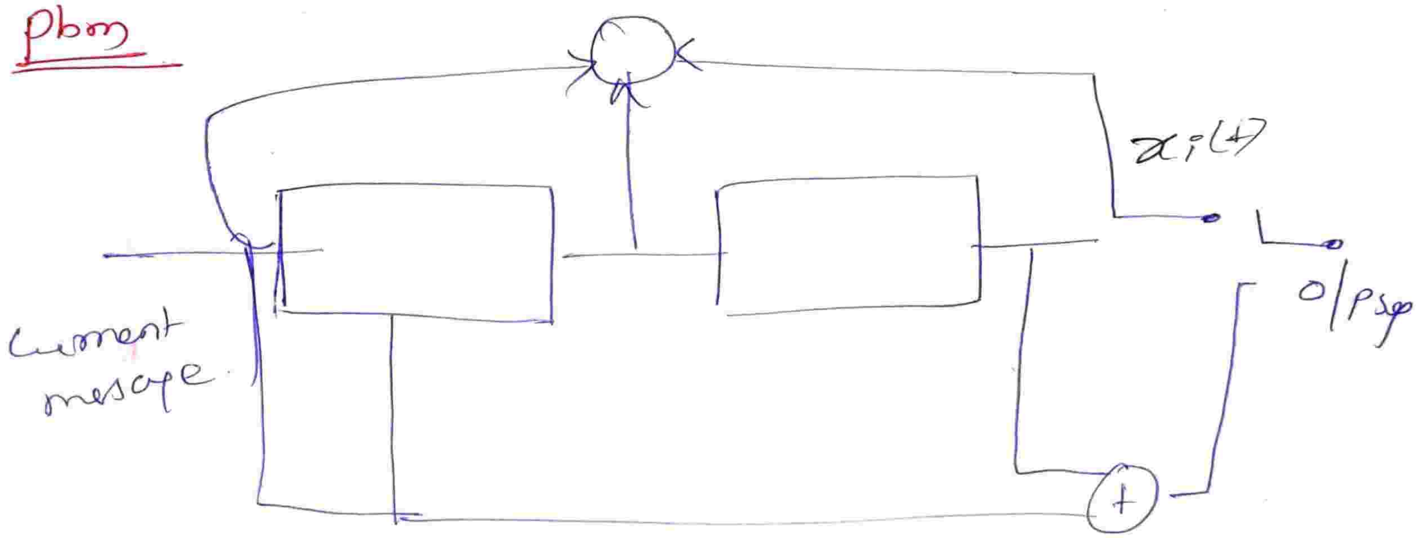
Code tree for message bit 1011

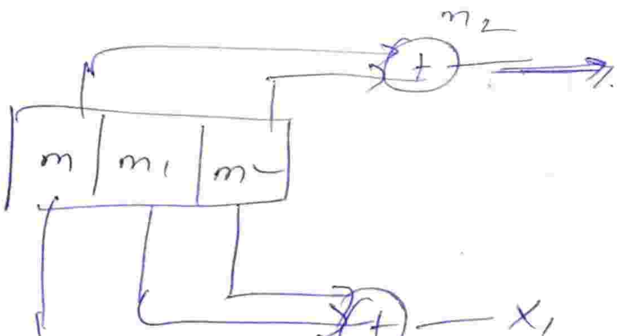
Time domain Approach:

$$x_1 = x_i(1) = \sum_{l=0}^{M} g_l^{(1)} m_{i-l} \qquad i = 0,1,2 \ldots$$

$$x_2 = x_i(2) \Rightarrow \sum_{l=0}^{M} g_l^{(2)} m_{i-l}$$

Pbm





1. Dimension of code
2. Code tree
3. Constraint length
4. Generating seq
5. O/p seq {10011}

pblm Important Terminologies

① Code rate

$R = \frac{k}{n}$

$\boxed{k = 1}$ ⇒ message bit.

msg bit = 1 (m) (single)

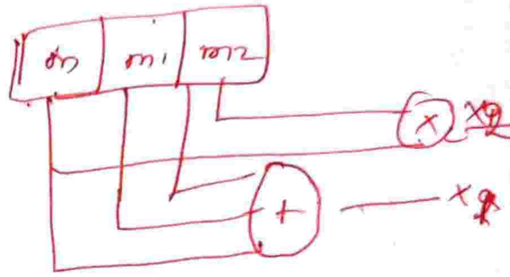
m_1, m_2 → previous va

② Constraint length   n ⇒ o/p seq ⇒ ②.

↓

No. of shift ⇒ 3.

in side the encoder (msg bit)



⇓

Code rate r = 1/2

Const. length ⇒ 3.

③ Dimension of the code.

(n, k) ⇒ (2, 1)

④ Generating sequence (g.)

$x_1 = m_1, m, m_2$

$x_2 = m, m_2$.

⑤ O/p sequence.

x_1 & x_2.

→ Time domain

→ Transfer domain Approach

$x_1 = \{1, 1, 1\}$

$g_i(l) = \{1, 1, 1\}$

$g_0^{(1)} \Rightarrow$ 1 rep connection of bit m

$g_1(1) =$ 1 rep connection

$$x_i^{(1)} = \sum_{l=0}^{M} g_i^{(1)}$$

g ↑ value
m ↓
↓ m = i value.
0, 1, 2, . m = i = 0, 1 ..

$M = 10011$

~~$i = 0$~~

~~$x_0^{(1)} = g_0^{(1)} . m_0$~~

To obtain o/p seq.

$m = \{m_0, m_1, m_2, m_3, m_4\} = \{10011\}$

$$x_i(1) = \sum_{l=0}^{M} g_l^{(1)} m_{i-l}.$$

$i = 0$ $$x_i(0) = \sum_{l=0}^{M} g_l^{(1)} m_{-l}$$

$= g_0^{(1)} m_0 . \Rightarrow 1 \times 1 = 1$

$m \rightarrow i$ val

g_2

if $i=1$

$$x_1(1) = g_0^{(1)} m_1 \oplus g_1^{(1)} m_0$$

$$= (1 \times 0 \oplus 1 \times 1) = 1$$

$i=2$

$$x_2(1) = g_0^{(1)} m_2 \oplus g_1^{(1)} m_1 \oplus g_2^{(1)} m_0$$

$$(1 \times 0) \oplus (1 \times 0) \oplus (1 \times 1) = 1$$

$i=3$.

$$x_4(1) = g_0^{(1)} m_4 \oplus g_1^{(1)} m_3 \oplus g_2^{(1)} m_2$$

$$= (1 \times 1) \oplus (1 \times 1) \oplus (1 \times 0)$$

$$= 0.$$

$i=4 \quad = 0$

$i=5 \quad = 0$

$i=6 \quad = 1$

$$x_i^{(1)} = \{1111\ 00\}$$

To obtain O/P due to adder 2

$$x_i(2) = \sum_{l=0}^{M} g_l^{(2)} m_{i-1}$$

i value

$a_2 \rightarrow$

If $i=0$, 1,

$i=1$; 0

$i=2$; 1

$i=3$; 1

$i=4$; 1

$i=5$ = 1

$i=6$ = 1. $x_i^{(2)} = \{1011111\}$

Encoded seq.

$x_i = \{11, 10, 11, 11, 01, 01, 11\}$.

Transform domain Approach:

Convolutional encoder is a linear time invariant finite state machine.

Impulse responses be represented by polynomials.

$$g_1^{(1)}(D) = g_0^{(1)} + g_1^{(1)}D + g_2^{(1)}D^2 + \cdots g_M^{(1)}D^M$$

$g_0^{(1)}, g_1^{(1)} \cdots g_M^{(1)}$ are the elements of the impulse response of the first path.

The variable 'D' denotes a unit delay operator.

|||ly

$$g^2(D) = g_0^{(2)} + g_1^{(2)}D^1 + \cdots g_M^{(2)}D^M$$

$g_0^{(2)}, g_1^{(2)}, \cdots g_M^{(2)}$ are the elements of the impulse response of the second path.

pbm

For the convolutional encoder, determine the o/p seq for message seq

(1) Draw the diagram of the ½ rate convolutional coder with generator polynomials $g^{(1)}(D) = 1+D$ and $g^{(2)}(D) = 1+D+D^2$, and complete the encoder o/p for i/p seq 101101

Given

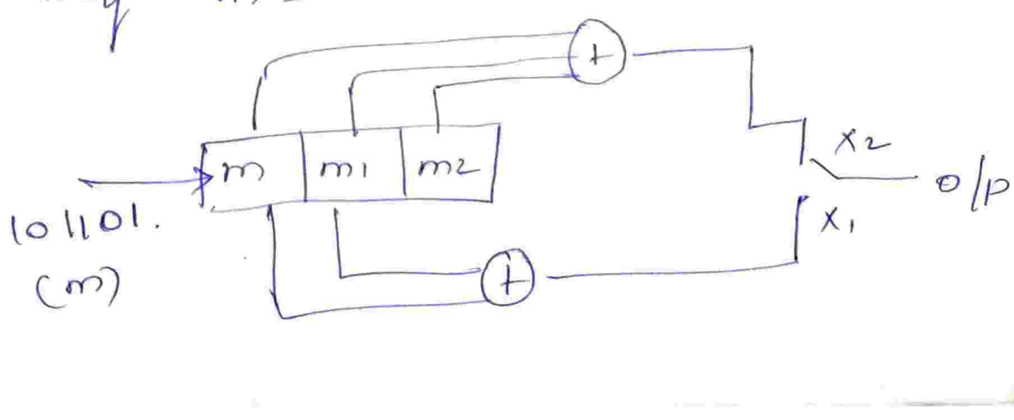
$r = 1/2 = k/n$ = $k \Rightarrow 1$ (msg)
$n = 2$ (o/p)

$g^{(1)}(D) = 1+D$ (Adder-1 o/p)

$g^{(2)}(D) = 1+D+D^2$ (Adder-2 o/p)

i/p seq $m = 101101$

Soln



Polynomial equation (Generating polynomials)

$g^{1}(D) = 1+D$

$g_1(p) = 1+p$

$g^{2}(D) = 1+D+D^2$

$g^{2}(p) = 1+p+p^2$

③ Polynomial eqn (message)

$m = 101101$

$m(p) = 1+(0\times p)+(1\times p^2)+(1\times p^3)+(0\times p^4)+(1\times p^5)$

$m(p) = 1+p^2+p^3+p^5$

④ Adder o/p

$(p^3+p^3)=0.$

mod. Adder −1 (o/p)

$x^{(1)}(p) = g^{(1)}p \cdot m(p)$

$= (1+p)(1+p^2+p^3+p^5)$

$1+p^2+p^3+p^5+p+p^3+p^4+p^6$

$$x^{(1)}(p) = 1 + p + p^2 + 0p^3 + p^4 + p^5 + p^6$$

$$x^{(1)} = (1\ 1\ 1\ 0\ 1\ 1\ 1)$$

mod Adder (2) o/p

$$x^{(2)}(p) = g^{(2)}p \cdot m(p)$$

$$= (1 + p + p^2)(1 + p^2 + p^3 + p^5)$$

$$= 1 + p^2 + p^3 + p^5 + p + p^3 + p^4 + p^6 + p^2 + p^4 + p^5 + p^7$$

$$= 1 + (p^2 + p^2) + (p^3 + p^3) + (p^5 + p^5) + p^6 + p^7$$

$$= 1 + p + 0p^2 + 0p^3 + 0p^4 + 0p^5 + p^6 + p^7$$

$(x_1, x_2 \ldots\ x_1, x_2 \ldots)$

$=$

$$x^{(2)}(p) \quad (1\ 1\ 0\ 0\ 0\ 0\ 1\ 1), \quad x_1(1)\ (1\ 1\ 1\ 0\ 1\ 1\ 1)$$

$$O/P = (1111100001010101110)$$

Code tree, Code trells and state diagram

Construct the code tree, Code trellis and state diagram

The parity check matrix of a particular (7,4) linear block code is given by

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

1. Find the generator matrix
2. List all code vectors.
3. What is the minimum distance b/w code vectors?
4. How many errors can be detected? and corrected

Given

$$H = [P^T : I]$$

$$P^T \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \Rightarrow P \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \qquad I \Rightarrow k =$$

$$G = [I_k : P_{k \times q}]_{k \times n} \qquad k = 4 \quad q = 3, \quad n = 7$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Syndro

Code vector

$k = 4 \Rightarrow$ msg bits

$C = MP$.

$$\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} m_1 & m_2 & m_3 & m_4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$c_1 = (1 \times m_1) \oplus (m_2 \times 1) \oplus (m_3 \times 1) \oplus (m_4 \times 0)$

$m_1 \oplus m_2 \oplus m_3$

$c_2 \Rightarrow m_1 \oplus m_2 \oplus m_4$

$c_3 \Rightarrow m_1 \oplus m_3 \oplus m_4$.

Code vector table.

msg bits $\rightarrow 4$.

check bits $\rightarrow 3$.

weight of codeword $\Rightarrow 3$.

d_{min} is $\cancel{\geq}$ 3

Syndrom = YH^T

$= EH^T$

Syndrom table $n = 7$, So error vector $E = 7$ bits.

$S = E \cdot H^T$

decode => Y.

Syndrome = [decoderalue] $[H^T]$

Syndrome = value → corresponding the error vector. →

For a systematic linear block code

$C = d_1 \oplus d_2 \oplus d_3$

$c = d_1 \oplus d_2$

$c_6 = d_1 \oplus d_3$

$q = 3$

$M = 3 = k =$

$q = n - k$

$n = q + k$

$= \equiv$

Construct G,

Error correction.

decode 101100 000110

3) For a

(3) For a Systematic linear block codes 3 parity check bits is c_4, c_5 and c_6 are given by

$c_4 = d_1 \oplus d_2 \oplus d_3 \Rightarrow$

$$\begin{array}{ccc} m_1 & m_2 & m_3 \\ m_1 & m_2 & 0 \\ m_1 & 0 & m_3 \end{array} \quad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$c_5 = d_1 \oplus d_2$

$c_6 = d_1 \oplus d_3$

Find the following (1) Calculate the generator Matrix

(2) Construct the code generator by using this matrix

(3) Determine the error correction capability

(4) Prepare the coding table (Syndrome table)

(5) Decode the received word 000110 and 000110.

Soln

$C = MP$

$G = [I_K : P$

$q = c$

$m = 3. \Rightarrow K$

$q = n - K$

$n = K + q$

$\boxed{n = 6} \rightarrow$ Error vector

(4) The parity check matrix of a (7, 4) Hamming code is given by

$$H \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Generator Matrix

Code vector for message [1011]

Draw the encoder Diagram.

Soln

(7, 4) $\quad n = 7 \quad k = 4$

$q = n - k = 3.$

G, $\quad$ message $m = 1011$

$$C = \begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

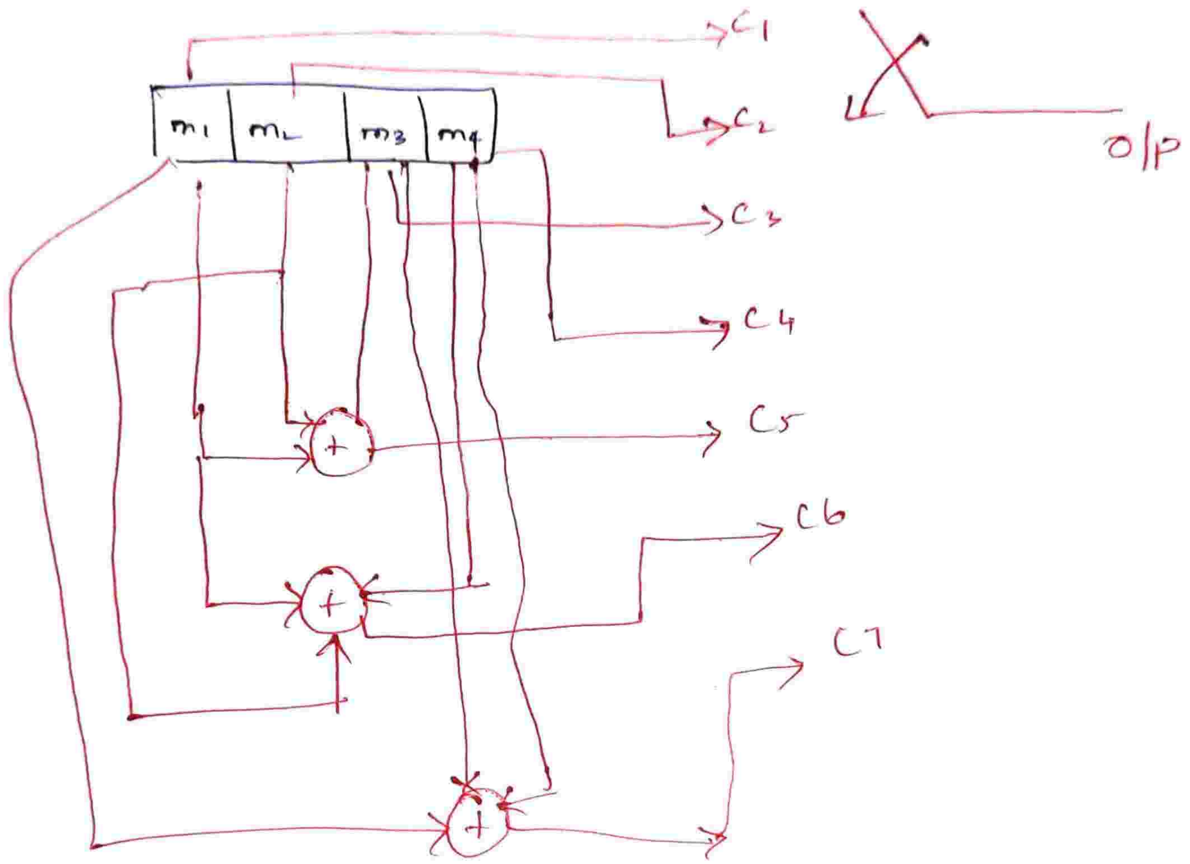
$C_1 = m_1 \quad C_2 = m_2 \quad C_3 = m_3 \quad C_4 = m_4$

$C_5 = m_1 \oplus m_2 \oplus m_3 \qquad C_6 = m_1 \oplus m_2 \oplus m_4$

$C_7 = m_1 \oplus m_3 \oplus m_4$

x)

Encoder dgm



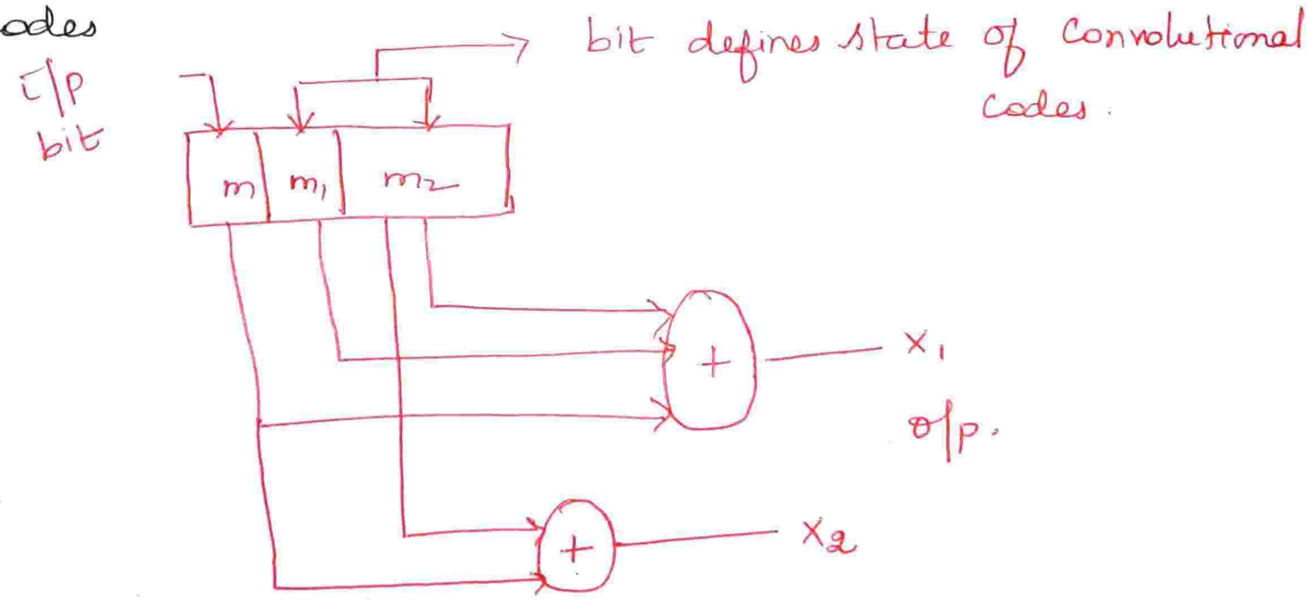
Code tree, Code Trellis and state diagram

→ Code tree indicates flow of the coded signal along the nodes of the tree.

→ Code tree is lengthy way of representing coding process.

pbms

code trellis and state diagram of convolutional codes

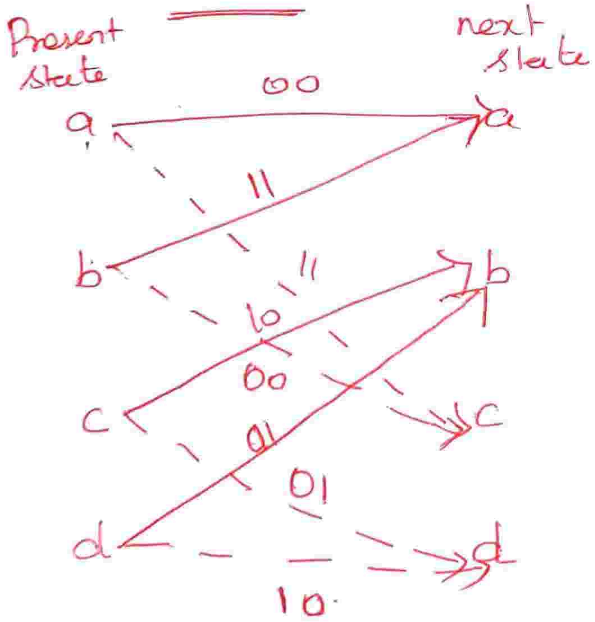


$$X_1 = m \oplus m_1 \oplus m_2$$

$$X_2 = m \oplus m_2$$

m	m_1	m_2	X_1	X_2	Current state	Next state
0	0	0	0	0	a	a
1	0	0	1	1	a	c
0	0	1	1	1	b	a
1	0	1	0	0	b	c
0	1	0	1	0	c	b
1	1	0	0	1	c	d
0	1	1	0	1	d	b
1	1	1	1	0	d	d

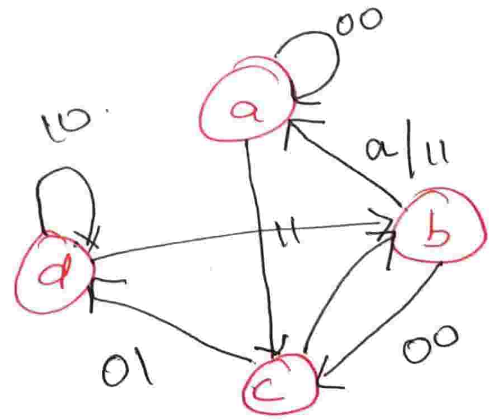
Code trellis



m_1	m_2	State
0	0	a
0	1	b
1	0	c
1	1	d

—————— i/p bit 0

- - - - - i/p bit 1



Viterbi Algorithm

→ It is a method of decoding Convolution Codes.

→ Here we use decoding algorithm.

Objectives:

To find the best path through the trellis that is closest to the received data bit sequence.

Step:1 – Trellis Encoder.

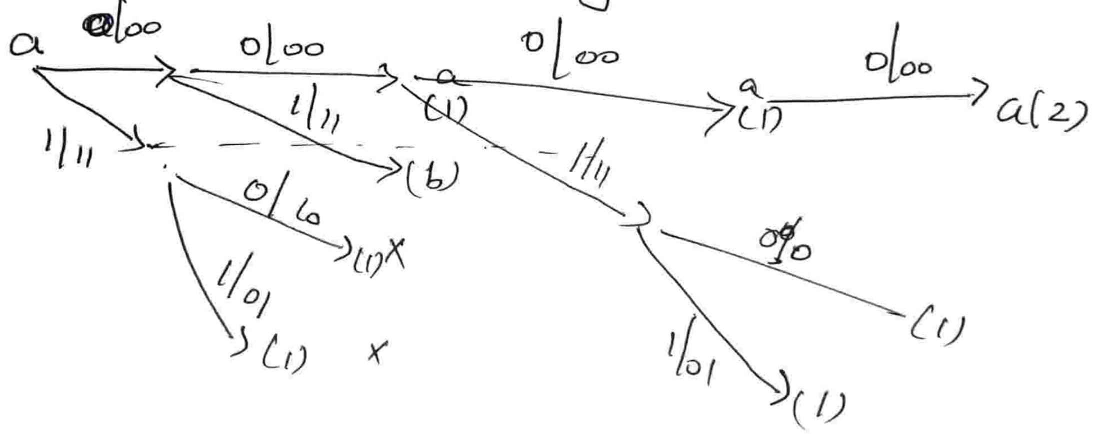
Coded seq → 01 001011

pbm

01 00 10 11

Metric → Calculate using Hamming distance.

Survivor path → path having Low minimum Hamming distance



Cyclic codes:

→ cyclic codes are subpart of Linear block codes It follows the following properties

① Linearity property:

→ If we have two code words c_i & c_j

then $c_p = c_i + c_j$

c_p → codeword.

② cyclic shifting

code word $= (c_1 \, c_2 \, \ldots \ldots \, c_n)$

After shifting left or Right by any no. of bits, redundant code should be a codeword.

→ ~~If~~

Eg:

{0000, 0110, 1001, 1111} is it cyclic code?

Soln

Check property of Linearity

$$\begin{array}{r} 0110 \\ 1001 \\ \hline \checkmark 1111 \end{array} \qquad \begin{array}{r} 0110 \\ 1111 \\ \hline \checkmark 1001 \end{array} \qquad \begin{array}{r} 1001 \\ 1111 \\ \hline \checkmark 0110 \end{array}$$

So it follows property of linearity.

Check property of shifting:

0110

0011 ← it is not a codeword.